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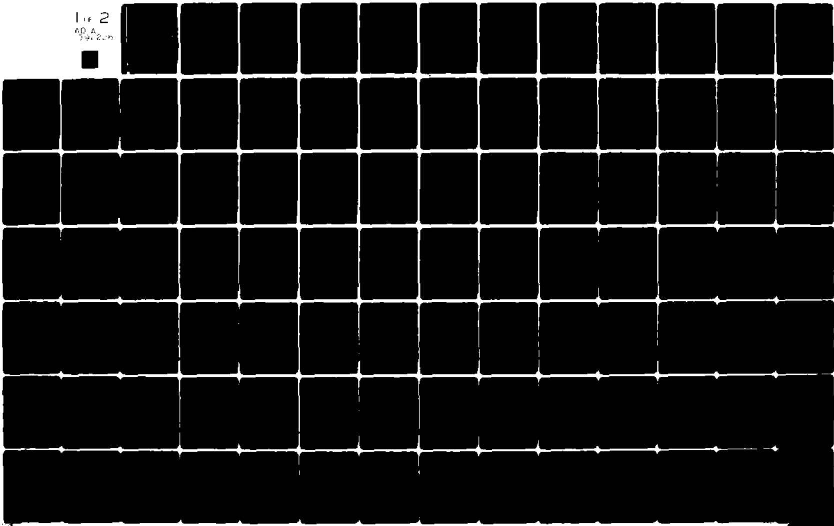

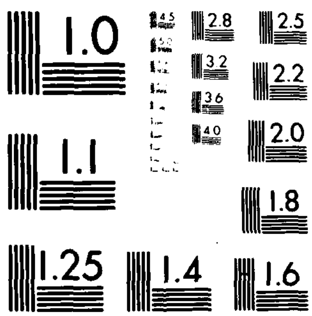

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EXTENSION OF PENCIL-OF-FUNCTIONS METHOD TO REVERSE-TIME PROCESSING WITH FIRST-ORDER DIGITAL FILTERS

BY

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In this presentation, the data signal is processed in reverse-time by a cascade of first order digital filters to yield a family of information signals. The Gram matrix of these information signals is shown to contain the essential information on the poles of the signal. The entire procedure of the application of pencil-of-function method is thus noniterative. Examples presented demonstrate (i) noiseworthiness in the representation problem when data are corrupted by noise, and (ii) the effectiveness of the method in the approximation problem.

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Comparison of the method with the maximum entropy method (or linear predictor) and the Prony method is also included.

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I. INTRODUCTION

Signal representation and approximation is basic to (a) time-domain extraction of singularities of a scatterer's field pattern. It is also useful in (b) bandwidth compression of signals, and (c) time-domain measurement and testing of networks/channels. This report discusses a unified approach to representing or approximating a given empirical signal by sum of exponentials, i.e., for finding the right hand side of

$$h_d(t) \cong h(t) = \sum_{i=1}^{n} W_i\, e^{\lambda_i t} \leftrightarrow H(s) = \sum_{i=1}^{n} \frac{W_i}{s-\lambda_i}$$

$$= \frac{\beta_{n-1}s^{n-1} + \beta_{n-2}s^{n-2} + .. + \beta_o}{s^n + \alpha_{n-1} + ... + \alpha_o}$$

or, equivalently, the right hand side of the sampled version

$$h_d(k) \simeq h(k) = \sum_{i=1}^{n} R_i(z_i)^k \leftrightarrow H(z) = \sum_{i=1}^{n} \frac{R_i}{(1-z_i z^{-1})}$$

$$= \frac{b_o + b_1 z^{-1} + ... + b_{n-1}z^{-n+1}}{1 + a_1 z^{-1} + ... + a_n z^{-n}}$$

The poles λ_i (or γ_i in z-domain) are either real, or they occur in complex conjugate pairs.

In the method described here, the data signal is processed in <u>reverse-time</u> by a cascade of first order digital filters -- each $\mu(z) = 1/(1-qz^{-1})$, to yield a family of information signals. The Gram matrix $\underset{\sim}{G}$ of these information signals is shown to contain the essential information on the denominator parameters of H(z). Specifically, it is shown that A(z) is determined as

$$A(z) = z^{-n} \left[\sum_{i=1}^{n+1} \sqrt{D_i}\, (z-1)^{n+1-i} \right] / \sqrt{D_1}$$

where D_i are the diagonal cofactors of the matrix $\mathcal{G}$. The numerator parameters are then determined using a least-squares fit, i.e., $\underline{b} = -\mathcal{P}^{-1}\underline{v}$, where $\mathcal{P}$ and $\underline{v}$ are defined in the paper.

The entire procedure is thus noniterative and computationally efficient. It is a further generalization of the method developed in [4]. Examples presented demonstrate (i) noiseworthiness in the representation problem when data are corrupted by noise and (ii) the effectiveness of the method in the approximation problem. Comparison of the method with the maximum entropy method (or linear predictor) and the Prony method is also included in the report.

The structure of the report is as follows. The fundamental results relating to the signal representation/approximation problem, through the use of reverse-time processing by first-order digital filters, are obtained in Chapter II. The important question of parallelism and orthogonality of the information signals is explored in Section III. Chapter IV gives the input description and user instructions for the program POF-FILTER which implements the method of Chapter II. Application examples as well as comparison with other methods, are given in Section V. Appendices A gives listing and description of program POF-FILTER and its routines. Appendices B and C contain the computer outputs relating to Example 1 and Example 2, respectively, of Section V.

SECTION II

FIRST-ORDER FILTER BASED PENCIL-OF-FUNCTIONS METHOD FOR MODELING IMPULSE RESPONSES

We shall be interested in modeling the impulse resonse [1]-[3]

$$y(k) = \sum_{\ell=1}^{n} R_\ell \, (z_\ell)^k \leftrightarrow \frac{B(z)}{A(z)} = \frac{b_1 z^{-1} + \ldots + b_n z^{-n}}{1 + a_1 z^{-1} + \ldots + a_n z^{-n}} \tag{1}$$

from its numerical data. Suppose a suitable K has been selected such that $y(k) = 0$ for $k > K$ (so that use of the upper limit K instead of ∞ on summations may be permitted). We define the reverse-time first-order filtered signals as

$$\begin{aligned}
y_1(k) &= y(k) \\
y_2(k) &= q y_2(k+1) + y_1(k) \\
&\ \ \vdots \\
y_N(k) &= q y_N(k+1) + y_n(k)
\end{aligned} \tag{2}$$

where $N=n+1$, and $y_i(K+1)=0$ for $i=1,2,..,N$. Further, $0 < q < 1$.

This family of signals, which we shall call information signals, exhibits the interesting property stated below.

Lemma 1

$$y_{i+1}(k) = \sum_{\ell=1}^{n} \frac{R_\ell}{(1-q\, z_\ell)^i} (z_\ell)^k \tag{3}$$

Proof: We prove this by induction. For $i=0$ the statement is trivially true since it is identical to (1) for this case. Assuming it to be true for $i-1$, let us proceed to prove it is true for i.

From (2)

$$y_{i+1}(k) = q y_{i+1}(k+1) + y_i(k) \tag{4}$$

which is readily shown to be equivalent to

$$y_{i+1}(k) = \sum_{\nu=k}^{\infty} q^{\nu-k}\, y_i(\nu)$$

$$= \sum_{\ell=1}^{n} \frac{R_\ell}{(1-q\ z_\ell)^{i-1}} \sum_{\nu=k}^{\infty} q^{\nu-k}(z_\ell)^{\nu} \qquad \text{(from induction hypothesis)}$$

$$= \sum_{\ell=1}^{n} \frac{R_\ell}{(1-q\ z_\ell)^{i-1}}\, z_\ell^k \sum_{\nu=k}^{\infty} q^{\nu-k}\, (z_\ell)^{\nu-k}$$

The result of equation (3) follows immediately by observing that

$$\sum_{\nu=k}^{\infty} q^{\nu-k}\, (z_\ell)^{\nu-k} = \frac{1}{(1-q\ z_\ell)}$$

This Lemma leads us to the crucial observation stated next.

<u>Lemma 2</u>

The set

$$(qz_m-1)y_2 + y_1,\ (qz_m-1)y_3 + y_2,\ \ldots\ ,\ (qz_m-1)y_N + y_n \tag{5}$$

is linearly dependent for $m=1,2,\ldots,n$ where z_m are the poles of the right hand side of (1).

Note: We have used the notation y_i to denote the sequence $\{y_i(k)\}$, $k=0,1,2,\ldots.$

Proof: In view of (3) we find

$$(qz_m-1)y_{i+1}(k) + y_i(k) = \sum_{\substack{\ell=1 \\ \ell\neq m}}^{n} \frac{q(z_m - z_\ell)R_\ell}{(1-q\ z_\ell)^i}(z_\ell)^k \tag{6}$$

for $i=1,2,\ldots,n$. Clearly, the sequences $(qz_m-1)y_{i+1}+y_i$, $i=1,\ldots,n$ each contain only n-1 modes $(z_\ell)^k$, $\ell\neq m$ hence are linearly dependent.

We can now apply the pencil-of-functions theorem of reference [4] to obtain the central theoretical result of this section.

We will call z_ℓ (see (1)) the poles of the impulse response, R_ℓ the corresponding residues, and $p_\ell = \{(z_\ell)^k\}$ the associated modes. Note that the poles occur in conjugate pairs whenever complex, as do the residues, since y is real.

Define the N x N dimensional Gram matrix (recall, N=n+1) [6]

$$F = \begin{bmatrix} \langle y_1,y_1\rangle & \cdots & \langle y_1,y_N\rangle \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \langle y_N,y_1\rangle & \cdots & \langle y_N,y_N\rangle \end{bmatrix}, \quad \langle y_i,y_j\rangle = \sum_{k=1}^{K} y_i(k)y_j(k) \tag{7a}$$

or, equivalently,

$$F = \sum_{k=1}^{K} \underline{f}(k)\underline{f}^T(k) \tag{7b}$$

where $\underline{f}^T(k) = [y_1(k)\ y_2(k)\ \ldots\ y_N(k)]$.

Theorem 1

The poles of the impulse response y(k) must satisfy the equation

$$\sum_{i=1}^{N} \sqrt{D_i}\ (qz-1)^{N-i} = 0 \tag{8}$$

where D_i are the diagonal cofactors of the Gram matrix F (defined in (7)

Proof: The theorem follows immediately upon application of the pencil of functions theorem (reference [4]) to the set (5).

Note that the denominator of transform of the impulse response is given by

$$A(z) = D_1^{-1/2}\ (qz)^{-n} \sum_{i=1}^{N} \sqrt{D_i}\ (qz-1)^{N-i} \tag{9}$$

This follows from (8) by dividing through by z^n and by normalizing the coefficients so that the leading coefficient becomes unity.

Determination of the denominator polynomial A(z) completes the first of the two steps in the decoupled pencil-of-functions method. The next step, finding the numerator coefficients in B(z), or equivalently finding the residues R_i, can be accomplished in two alternative ways.

The first method consists in solving for the residues from the equation[1]

$$\begin{bmatrix} \frac{1}{1-qz_1} & \frac{1}{1-qz_2} & \cdot\ \cdot & \frac{1}{1-qz_n} \\ \frac{1}{(1-qz_1)^2} & \frac{1}{(1-qz_2)^2} & \cdot\ \cdot & \frac{1}{(1-qz_n)^2} \\ \vdots & \vdots & \vdots\ \vdots & \vdots \\ \frac{1}{(1-qz_1)^N} & \frac{1}{(1-qz_2)^N} & & \frac{1}{(1-qz_n)^N} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_n \end{bmatrix} = \begin{bmatrix} y_2(0) \\ y_3(0) \\ \vdots \\ y_N(0) \end{bmatrix} \qquad (10)$$

This equation follows from (3) upon setting k=0 and letting i range from 1 to n. Clearly, the use of this equation requires that the poles z_i be determined from the denominator A(z) by use of a root-finding routine. This requirement is of no consequence if the final answers are needed in the s-domain, for conversion to s-domain involves finding the roots of A(z) anyway.

The alternative approach is to find the optimum least-squares numerator coefficients (given the denominator of (9)) through the equation

---

1 The equation corresponding to i=0 is ignored in formulating (10) because of the relatively poor signal/noise statistics of $y_1(0)$ (compared to, say, $y_2(0)$) when the impulse response data contain additive wideband noise.[2] This statement assumes that the bandwidth of y(k) is much smaller than that of the additive noise.

$$
\begin{bmatrix}
<w_1, w_1> & <w_1, w_2> & \cdots & <w_1, w_n> \\
<w_2, w_1> & <w_2, w_2> & \cdots & <w_2, w_n> \\
\cdot & \cdot & \cdots & \cdot \\
\cdot & \cdot & \cdots & \cdot \\
<w_n, w_1> & <w_n, w_2> & \cdots & <w_n, w_n>
\end{bmatrix}
\begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ b_n \end{bmatrix}
=
\begin{bmatrix} <y, w_1> \\ <y, w_2> \\ \cdot \\ \cdot \\ <y, w_n> \end{bmatrix}
\tag{11}
$$

where w_i denotes the impulse response of $z^{-i}/A(z)$. Note that $w_i(k)=w(k-i)$ where $w(k)$ is the impulse response (i.e., inverse z-transform) of $1/A(z)$. All inner products are summed from $k=0$ to K.

Discussion

Equations (9) and (11) have been implemented in a computer program "POF-FILTER" written in FORTRAN IV. The program is presented in Section III.

The idea of reverse-time integration was proposed by Çarr in [7] and Jain in [8]. Here, we have generalized the concept of reverse-time processing to the case of first-order filter processing. Note that the first order filter $1/(1-qz^{-1})$, used above, encompasses integration; just let $q=1$.

It should be borne in mind that the approach developed above is applicable to impulse responses only. With some effort, it may be modified for use to step responses and square-pulse responses. For more general inputs, however, one must use the coupled approach dicussed in [9] (which of course involves greatly increased computations, e.g., the Gram matrix involved is of an order twice as high as in the decoupled procedure).

The decoupled approach can be used only if the data is of sufficient length K such that $y(k>K)=0$ for all practical purposes.

The reverse-time processing of $y(k)$ through the first order filters can be interpreted as forward-time processing of the signal $h(k)=y(K-k)$ through the same filters.

The use of square-roots of cofactors of the Gram matrix is analogous to the use of square-root factorization [10] of the Gram matrix. Attendant advantages are therefore expected to be realized. A more detailed analysis of this connection will be discussed elsewhere.

The transfer function of the first order filter used in equation (2) is $\mu(z)= 1/(1-qz^{-1})$. Instead, we could use filters with transfer funnction $\mu_1(z)= (1-q)/(1-qz^{-1})$; these filters have a d.c. gain equal to unity and the ratio of the output power to the input power is a direct measure of the extent of the rejection of higher frequencies. Equation (9) remains valid even when these unity d.c. gain filters are used.

Note that the first-order filtering (in reverse time) is achieved in (2) recursively, without the need to carry out discrete convolution.

SECTION III

PARALLELISM AND ORTHOGONALITY OF INFORMATION SIGNALS

Here we consider the important matter of parallelism and orthogonality of the information signals. In the last section we processed the signal $h(k) = y(K-k)$ through first order filters $\mu_1(z) = (1-q)/(1-qz^{-1})$. This is depicted in Fig. 1. Note that forward time processing of $h(k)$ is equivalent to reverse-time processing of $y(k)$. Because of familiarity with forward-time processing we will carry out the discussion below in terms of the signal $h(k)$. Also note that $h_i(k) = y_i(K-k)$, $i=1,2,\ldots,N$. Finally, we remark that the Gram matrix and the properties of parallelism/orthogonality of the two families of signals $h_1, h_2, \ldots, h_N$ and $y_1, y_2, \ldots, y_N$ are identical. Indeed, $\langle h_i, h_i \rangle = \langle y_i, y_i \rangle$ and $\langle h_i, h_j \rangle = \langle y_i, y_j \rangle$ for $i,j=1,2,\ldots,N$.

$$h(k)=h_1(k) \longrightarrow \mu_1(z) \xrightarrow{h_2} \mu_1(z) \xrightarrow{h_3} \cdots \longrightarrow \mu_1(z) \xrightarrow{h_N}$$

Fig. 1. Generation of information signals by use of first-order filters.

The magnitude (vs. frequency) characteristic of the first order filter $\mu_1(z)$ is shown in Fig. 2. The cutoff frequency (- 3dB point) ω_c is related to the parameter q as [5]

$$\omega_c = \frac{1}{\Delta} \; Ln\left(\frac{1}{q}\right) \qquad (12)$$

where Δ is the sampling interval. Defining Ω as the normalized frequency (i.e., the ratio ω/ω_s where ω_s is the sampling frequency in radian/s) we can express the normalized cutoff frequency Ω_c as

$$\Omega_c = \frac{1}{2\pi} \, Ln(1/q) \qquad (13)$$

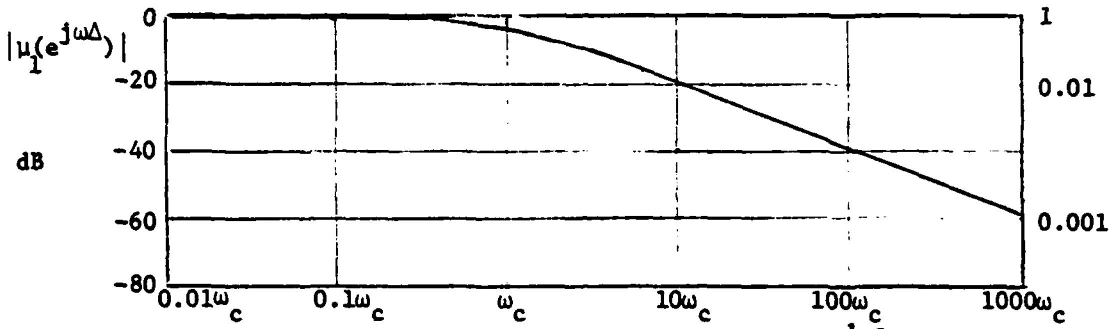


Fig. 2. Magnitude characteristic of $\mu_1(z)=\frac{1-q}{1-qz^{-1}}$

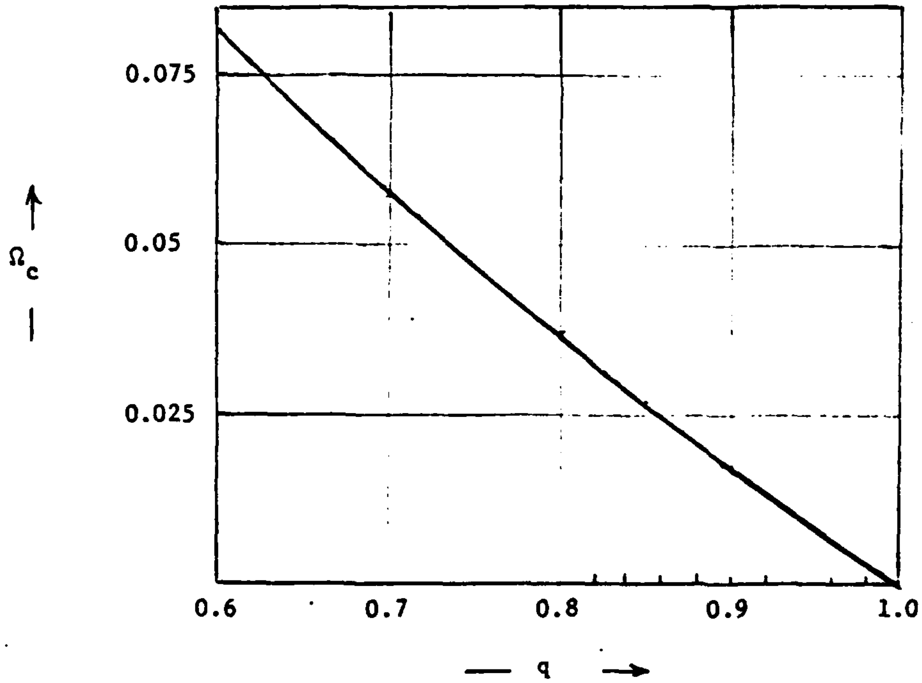


Fig. 3 Normalized cutoff frequency vs. parmeter q

The normalized cutoff frequency Ω_c is plotted against the filter parameter q in Fig. 3.

To analyze the parallelism/orthogonality (PO) properties of the information signals let us define the <u>connection filters</u>

$$M_0(z) = 1$$
$$M_i(z) = (\mu_1(z))^i, \qquad i=1,2,\ldots,n \tag{14}$$

so that we may write

$$H_{i+1}(z) = M_i(z)\ H_1(z) \tag{15}$$

Note that $M_i(z)$ is an i-pole filter with a pole of multiplicity i at z=q. The magnitude characteristic of the filters $M_0,\ldots,M_4$ are shown in Fig. 4 for q=0.8.

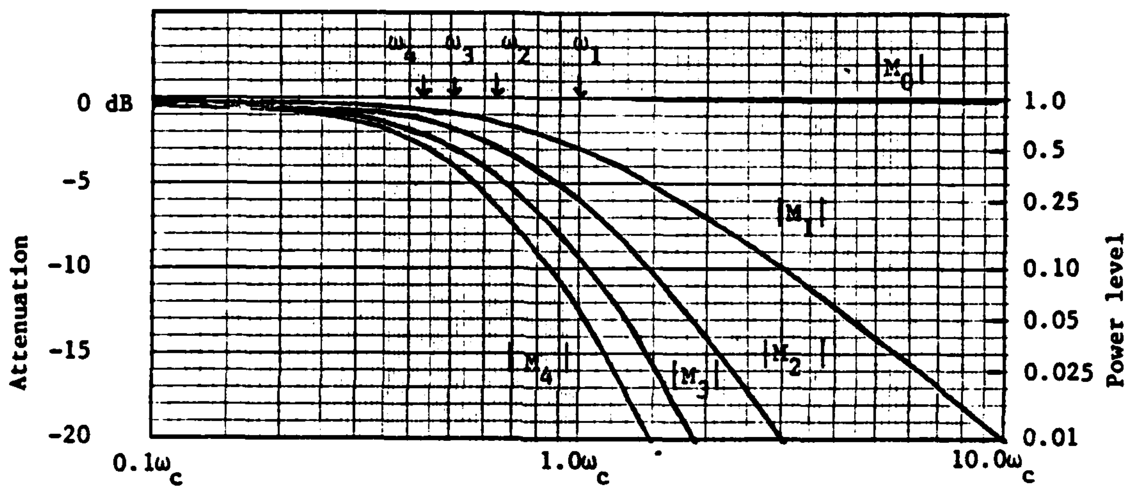


Fig. 4. Magnitude vs. Freq. of connection filters

We now present an approximate, and somewhat heuristic, analysis of the PO properties of the family of information signals.

Approximations -

We approximate the magnitude characteristics of the connection filters M_i as shown in Fig. 5a.

The phase properties of M_i will be ignored (assumed identically zero)

It is then possible to write

$$M_i(e^{j\omega\Delta}) = L_i(\omega) + \ldots. + L_n(\omega) \tag{16}$$

where $L_i(\omega)$ are the bandpass filter characteristics shown in Fig. 5b. Further, (15) and (16) together yield

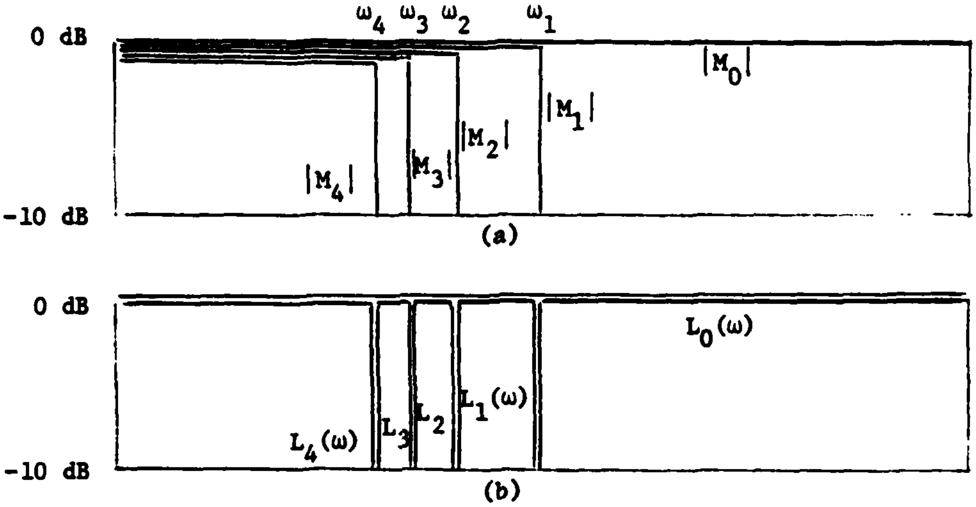


Fig. 5. Idealized connection filters and BP constituents.

$$H_{i+1}(e^{j\omega\Delta}) = (L_i(\omega) + \dots + L_n(\omega))\, H_i(e^{j\omega\Delta}) \tag{17a}$$

$$= \Phi_i(\omega) + \dots + \Phi_n(\omega) \tag{17b}$$

where

$$\Phi_i(\omega) = L_i(\omega)\, H_i(e^{j\omega\Delta}) \tag{18}$$

Clearly, $\Phi_i(\omega)$, $i=0,1,\dots,n$ are orthogonal. Indeed,

$$\int_{-\omega_s/2}^{\omega_s/2} \Phi_i(\omega)\, \Phi_j^*(\omega)\, d\omega = \int_{-\omega_s/2}^{\omega_s/2} |H_1(\omega)|^2\, L_i(\omega) L_j^*(\omega)\, d\omega$$
$$= 0 \quad \text{for } i \neq j \tag{19}$$

because $L_i(\omega)=0$ wherever $L_j(\omega)\neq 0$ and vice-versa. Let us state this in the time domain as

$$\langle \phi_i, \phi_j \rangle = \sigma_{ii}\ \delta_{ij} \tag{20}$$

where δ_i is the Kronecker delta and $\{\phi_i(k)\} = \mathcal{F}^{-1}\ \Phi_i(\omega)$.

We can summarize the above discussion as follows.

<u>Theorem 2</u>

The information signals are approximately tri-orthogonal.

Proof: From (18) and (20) we have

$$
\begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ \cdot \\ \cdot \\ h_N \end{bmatrix} \simeq \begin{bmatrix} 1 & 1 & 1 & \cdot\ \cdot & 1 \\ 0 & 1 & 1 & \cdot\ \cdot & 1 \\ 0 & 0 & 1 & \cdot\ \cdot & 1 \\ \cdot & \cdot & \cdot & \cdot\ \cdot & 1 \\ \cdot & \cdot & \cdot & \cdot\ \cdot & \cdot \\ 0 & 0 & 0 & 0\ 0 & 1 \end{bmatrix} \begin{bmatrix} \phi_0 \\ \phi_1 \\ \phi_2 \\ \cdot \\ \cdot \\ \phi_n \end{bmatrix} \tag{21}
$$

where $\phi_0, \ldots, \phi_n$ are a set of orthogonal signals. The approximation sign arises because of the assumptions made earlier.

Now reversing the time-indices about $k=K$, we have

$$
\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \cdot \\ \cdot \\ y_3 \end{bmatrix} \simeq \begin{bmatrix} 1 & 1 & 1 & \cdot\ \cdot & 1 \\ 0 & 1 & 1 & \cdot\ \cdot & 1 \\ 0 & 0 & 1 & \cdot\ \cdot & 1 \\ \cdot & \cdot & \cdot & \cdot\ \cdot & 1 \\ \cdot & \cdot & \cdot & \cdot\ \cdot & \cdot \\ 0 & 0 & 0 & 0\ 0 & 1 \end{bmatrix} \begin{bmatrix} \psi_0 \\ \psi_2 \\ \psi_3 \\ \cdot \\ \cdot \\ \psi_n \end{bmatrix} \tag{22}
$$

where, by definition $\psi_i(k) = \phi_i(K-k)$. Since

$$
\begin{aligned} \langle\phi_i, \phi_j\rangle &= \langle\psi_i, \psi_j\rangle \\ &= \sigma_i\, \delta_{ij} \end{aligned} \tag{23}
$$

the assertion of the theorem is proved.

Note that the weights σ_i can be calculated (in view of (18)) as

$$
\sigma_i = \int_{-\omega_s/2}^{\omega_s/2} \left| L_i(\omega)\, H_1(e^{j\omega\Delta}) \right|^2 d\omega
$$

Else, they may be calculated in the time domain via (20).

Note that (22) may be writen more compactly as

$$\underline{y} = U \underline{\psi} \tag{25}$$

where $\underline{y} = [y_1, y_2, \dots, y_N]^T$, $\underline{\psi} = [\sigma_0, \sigma_1, \dots, \sigma_n]^T$ and U is the upper triangular matrix of 1's. Secondly, we observe that the Gram matrix of the information signals can be written as

$$F = \langle \underline{y}, \underline{y}^T \rangle \tag{26}$$

where the inner-product is taken over each term of the matrix $\underline{y}\,\underline{y}^T$. In terms of the approximate tri-orthogonal characterization in (22), or (25), we have

$$F \simeq U \langle \underline{\psi}, \underline{\psi}^T \rangle U^T$$

$$= U \Sigma U^T \tag{26}$$

where Σ is the n+1 dimensional diagonal matrix diag$\{\sigma_0, \sigma_1, \dots, \sigma_n\}$.

The lemma below summarizes the PO properties of the information signals.

<u>Lemma 3</u>

The correlation coefficient of a pair of information signals y_i and y_j, $j > i$ is

$$\rho_{ij} \simeq \sqrt{\frac{\sigma_{j-1} + \dots + \sigma_n}{\sigma_{i-1} + \dots + \sigma_n}}, \qquad i,j=1,\dots,n+1 \tag{27}$$

Proof - The relation follows readily from the approximate tri-orthogonal characterization of the information signals. Using (22) and (23), we have

$$\rho_{ij} \simeq \frac{\sigma_{j-1} + \dots + \sigma_n}{\sqrt{\sigma_{i-1} + \dots + \sigma_n}\sqrt{\sigma_{j-1} + \dots + \sigma_n}}$$

which is the same as (27).

Discussion

In any system identification technique it is desirable to have basis functions that differ significantly from each other. Ideally, they should be orthogonal. By varying the parameter q between 0 and 1, it is possible to generate sets of basis functions that vary between a set whose elements are almost identical to approximately a set whose elements are orthogonal.

If $q \approx 0$ (recall that q is the z-domain pole of the first order filters $\mu_1(z)$), then the connection filters M_i all have a nearly all-pass characteristic, and as a result $\sigma_i \approx 0$ for $i \neq n$. Thus σ_n dominates all other σ_i and we have

$$\rho_{ij} \approx 1$$

This is undesirable; however, such is indeed the case when unit-delays are employed such as in methods like Prony, Linear Predictive analysis, Maximum-Entropy method, etc. [1]-[2]. The strong correlation between the basis functions leads to numerical difficulties.

On the other extreme, $q=1$ results in pure (digital) integration of $y(k)$, in reverse-time, for generation of the information signals. In this case the analysis of PO properties developed above is not applicable. Therefore, consider q close to one (from the left; e.g., $q = 0.99$. Now the cutoff frequencies $\omega_1, \omega_2, \ldots, \omega_n$ become crowded near the zero frequency. Hence $\sigma_i \approx 0$ for $i \neq 0$ and σ_0 dominates all other σ_i. Therefore, $y_2, \ldots, y_N$ have very small energy (thus very small fraction of signal information), and they are nearly orthogonal to y_1.

Intermediate values of q lead to other useful sets of basis functions.

Example

Consider that the signal $y(k)$ has flat low-pass spectrum from $\Omega = 0$ to 0.05 with amplitude level 10. Then $q = 0.8$ and $n = 4$ lead to the values

$$\sigma_0 = 1.45,\ \sigma_1 = 1.28,\ \sigma_2 = 0.46,\ \sigma_3 = 0.31,\ \sigma_4 = 1.5$$

from which the correlation coefficients can be computed readily. For example $\rho_{23} = 0.893$, and $\rho_{14} = 0.548$.

SECTION IV

PROGRAM DESCRIPTION

The program POF-FILTER is a high accuracy FORTRAN IV program designed to implement the decoupled pencil-of-functions method. Specifically, it models the impulse response of a finite-order linear system by processing the given data through a cascade of first-order linear filters in reverse-time. Some of the features of the program are stated below.

* Decoupled denominator and numerator determination. This permits fairly high order models to be determined, since the order of the Gram matrix is n+1 where n is the model order (or n+2 when data bias is also to be estimated. In contrast, the coupled procedure requires the use of a 2n+2 order Gram matrix (or 2n+3 when bias is also to be estimated)
* First-order filter, rather than pure integrators, are used for generating the family of information signals. This results in a nearly tri-orthogonal set of signals which result in a better conditioned Gram matrix than with the use of pure integrators.
* Bias correction option. Data bias can be estimated and thereby more accurate estimates for the model transfer function parameters obtained.
* Noise correction option. A preliminary routine for estimation of noise effect,and correction thereof, has been included. Theoretical work and testing/improvement of this routine remains to be done.
* Direct transmission option. Structural correctness of the model in either the presence or absence of the direct transmission term is preserved by exercising this option.
* Results of modeling are obtained in the z-domain and on an optional

basis in the s-domain also.

* Simulation option. The data modeled may be laboratory test data, or one may generate an impulse response within the program by specifying it either in the form $\sum_{k=1}^{n} R_\ell (z_\ell)^k$ or by the z-domain transfer function $H(z)=(b_0+b_1z^{-1}+ \ldots +b_nz^{-1})/(1+a_1z^{-1}+ \ldots +a_nz^{-n})$.

  In the simulation mode, a desired amount of bias and/or additive noise may be incorporated for test purposes.

  In the laboratory-data case, a preliminary bias removal and a data scaling procedure (to maximize the effectiveness of the algorithm) has been incorporated.

* The routine which finds the cofactors of the Gram matrix of the information signals has been optimized by incorporating a scaling and corresponding descaling stages.

* For comparison purposes the program provides the option to use two other modeling techniques, the linear predictor and the Prony method. The latter can be the classical Prony, which uses 2n+2 data points, or the least-squares Prony.

  The provision of these methods within this single program is an essential step toward the evaluation of the pencil-of-functions method against other benchmark methods.

The input data cards on the subsequent pages give a description of all input variables, and in so doing provide an understanding of the program use.

INPUT DATA CARDS

CARD # 1 The first card is a title card.

CARD # 1 The first card is a title card. Columns 1 through 70 are available for alphanumeric title.
This title is reproduced in the output

CARD # 2 Another title card (columns 1-70). Not reproduced in output

CARD # 3 Another title card (columns 1-70). Not reproduced in output

CARD # 4 First option card

Variable Name (Format)	Description	Columns	Preferred/default Value
NPT (I4)	Number of signal points used in analysis/modeling	1-4	-
IXX, IYY (2I2)	Blank (unused)	5-8	-
N (I2)	Model order	9-10	-
ISIM (I2)	Simulation mode option ISIM = -1 Real (laboratory) data in integer format (10I5) = 0 Real (laboratory) data in real format (10F8.6) = 1 simulation from digital transfer function = 2 simulation from sums of exponential*sinusoid form (specified in continuous-time domain)	11-12	-
NCOMP (I2)	Number of terms in the sums of exponential*sinusoid form	13-14	-
IPLT (I2)	Plot option IPLT = 0 Plot routine not called = 1 Plot of the original data and model response obtained = m same as 1 except that every mth point of the signals are plotted (e.g., with m=2 alternate points are plotted)	15-16	
NNPT (I4)	Number of signal points read or through simulation. NNPT should be greater than or equal to NPT	17-20	NPT

YYYY (F10.0)	Blank (unused)	21-30	
DT (F10.0)	Sampling interval	31-40	
BIAS (F10.0)	Bias to be added to data. This is for use in the simulation modes (ISIM=1 or 2) when it is desired to study the effect of bias on data	41-50	
ANBIAS (F10.0)	Number of points used for a preliminary estimate of bias. That is, NBIAS= Integer(ANBIAS) number of points from the right are used to find a crude estimate of bias; this crude bias is subtracted from data. This is useful only when real data is analyzed (ISIM= -1 or 0), and ignored when simulation data is generated.	51-60	Leaving this blank results in the use of 20% dta from the right for crude bias.
VAR (F10.0)	Variance of noise to data. Must be used only in the simulation mode (ISIM=1 or 2)and should be left blank when real data is analyzed.	61-70	
CARD # 5.1-5.n	If ISIM= -1 or 0 these cards contain real data	1-50 if ISIM=-1 1-80 if ISIM=0	

If ISIM= 1 these cards contain (in (5F10.0) format the coefficients of the z-domain transfer function; first denominator coefficients, and on succeeding card(s) the numerator coefficients.

If ISIM= 2 these cards contain (in (5F10.0) format the coefficients of the exponential*sinusoid terms; one such term on each card. Each card contains a) the weighting coefficient b) the exponent (real), c) the radian frequency of the sinusoid, and d) the phase of the sinusoid. If the phase is zero, the sinusoid is a sine wave with zero value at k=0.

Example: If $y(t)= e^{-t} + 7e^{-3t} \cos(2t)$ is to be simulated, then ISIM=2, NCOMP=2 and these cards are as follows

```
+1.00000  -1.00000
+7.00000  -3.00000  +2.00000  +1.57079
```

CARD # 6 Subtitle card. This alphanumeric subtitle is reproduced in the output

CARD # 7 Second option card. This numeric information is also reproduced on the output (for user convenience)

Variable Name (Format)	Description	Columns	Preferred/Default Value
IPR (I2)	Print option. Increasing value results in more printing. Use 0, 1 or 2 for normal use. Values 3, 4 and 5 are useful for diagnostic purposes, or when resut-s of intermediate computations are needed (for example, the first-order filtered signals, i.e., the information signals are printed only if IPR.GE.4)	1-2	0
IZTS (I2)	z-domain to s-domain conversion option. IZTS = -1 conversion to s-domain is not performed = 0 conversion performed; only the poles in s-domain printed = 1 conversion performed; in addition to poles in s-domain the s-domain denominator and numerator also printed.	3-4	
IREM (I2)	Number of coefficients in the numerator (of the z-domain model), counted from the right, and set to zero	5-6	0
ISPN	Method option ISPN = -1 pencil-of-functions method employed = 1 pencil-of-functions method employed; noise added to simulated data. Must be used only when ISIM=1 or 2 = 0 Analysis of noise only =-2 Covaria ce equations used (for LPC or Prony) =-3 Autocorrelation equations used (for LPC or Prony)	5-6	 Warning: Do not use For LPC set IREM=19 For LPC set IREM=19

IFIX (I2)	Noise correction option (under development - ignore)	9-10	
NFIX (I2)	Auxiliary parameter for use with IFIX - ignore	11-12	
IBIAS (I2)	Bias extraction option IBIAS = 0 Bias extraction not exercised = 1 bias extracted	13-14	Warning: Do not use values other than 0 or 1
IBØ (I2)	Direct transmission term option IBØ = 0 Constrains $b_0=0$ in numerator determination IBØ = 1 Model assumes direct transmission is present, and b_0 is determined together with other coefficients	15-16	
MNPT (I4)	Controls the number of points used in the computation of error, and for printing (when IPR.GE.2) and plotting	17-20	NPT
QI (F10.0)	z-domain pole of the first order filter (i.e., the parameter q of Section I)	21-30	suggest QI=0.8
DFAC (F10.0)	Auxiliary variable for use with IFIX (under development)	31-40	

SECTION V

APPLICATION EXAMPLES

Two examples will be presented in this section. The first deals with a simulated noisy signal, $x(k) = y(k) + w(k)$ where $y(k)$ is the response of a third order transfer function and $w(k)$ is a zero mean noise component. The performance of the pencil-of-functions method, with and without Gram matrix enhancement, is compared with that of other methods [1]-[3]. The second example pertains to the transient response of a conducting pipe tested at the ATHAMAS-I EMP simulator. These examples demonstrate the effectiveness of the pencil-of-functions method as a practical modeling technique.

<u>Example 1</u>

Let

$$y(k) \quad \leftrightarrow \quad \frac{1 - 1.92z^{-1} + z^{-2}}{1 - 2.68z^{-1} + 2.476z^{-2} - 0.782z^{-3}}$$

where the sequence y is truncated at K=99. The signal under test is obtained as

$$x(k) = y(k) + w(k)$$

where $w(k)$ is a zero mean, uncorrelated sequence with standard deviation equal to 0.0316. The energies of the signal and noise components are 1.82 and 0.1, respectively, hence the sequence under test has a signal-to-noise ratio of 12.6 dB. The numerical data and a plot of the signal under test are given in Appendix B.

The signal under test was analyzed by the following methods [2].

1. Pencil-of-functions method
   a) without enhancement of the Gram matrix
   b) with enhancement of the Gram matrix

2. Linear predicive coder (LPC)
   a) using autocorrelation equations
   b) using covariance equations

3. Prony's method [11]
   a) using autocorrelation equations
   b) using covariance equations [3]

The computer output for these six runs is given in Appendix B. Here we summarize the fractional energy error, defined as

$$\nu = \frac{\sum_{k=0}^{K} (y(k) - \hat{y}(k))^2}{\sum_{k=0}^{K} y^2(k)} \tag{28}$$

where $\hat{y}(k)$ is the impulse response of the model. Note that in simulation mode the model response can be, and is, compared with the true signal. When real data is tested the true signal is not available, hence $x(k)$ must be used in this equation instead of $y(k)$.

For the present example the true signal is available, hence the fractional energy error is computed by exact application of (28). The values for the various methods are tabulated in Table 1.

TABLE 1

Method	Fractional energy error
Pencil-of-functions	0.0167
Pencil-of-functions with enhancement	0.0016
Linear Predictor (autocorrelation eqn)	0.1444
Linear Predictor (covariance eqn)	0.1567
Prony (autocorrelation eqn)	0.1431
Prony (covariance eqn)	0.1397

The model responses are compared with the true signal $y(k)$ in Figures 6 through 11. The reader is cautioned that the solid line in each of these figures represents the true signal, and not the noisy signal under test. The latter is shown in Fig. B2 in Appendix B. The dotted line in each of these figures represents the model response.

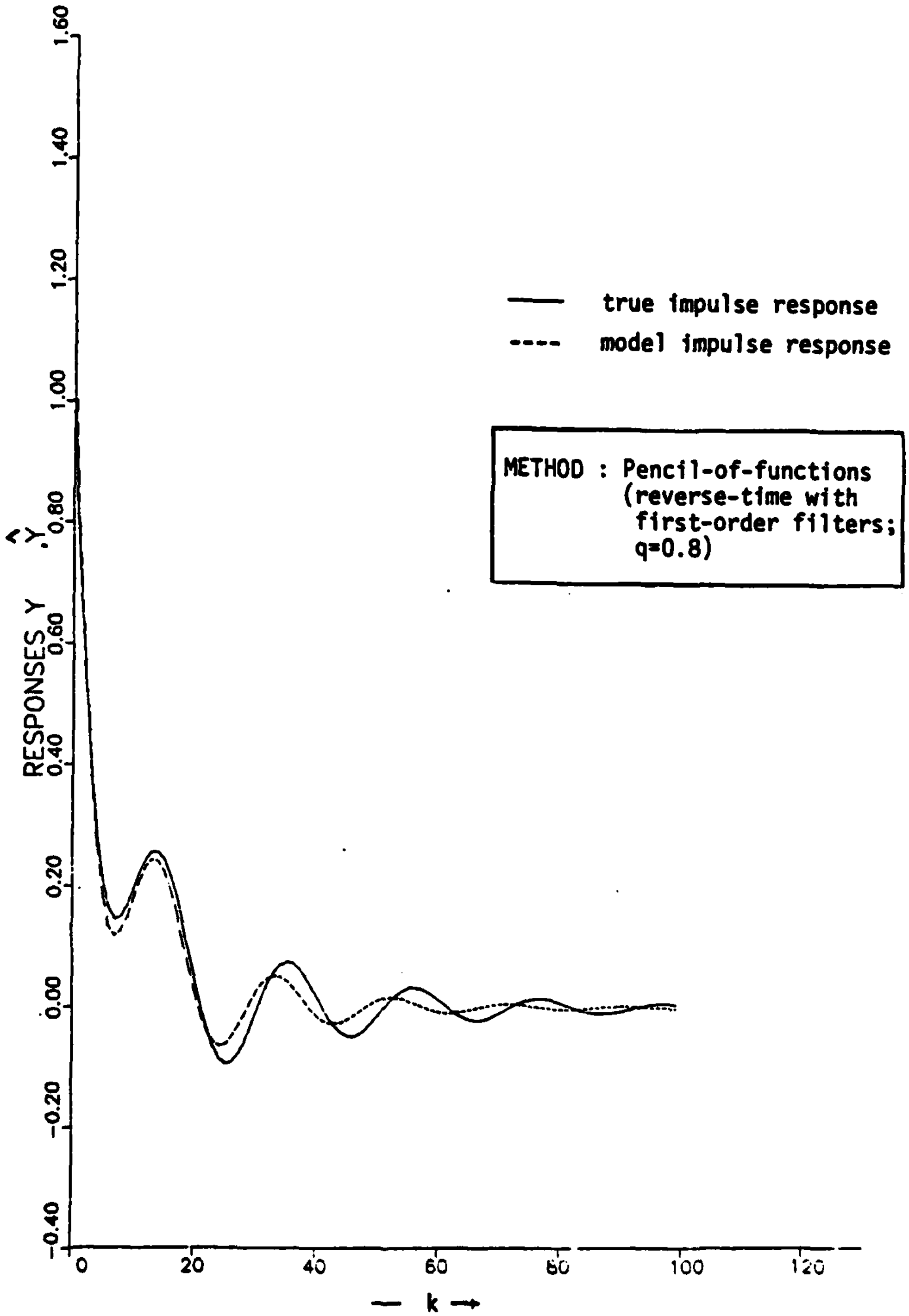


Fig. 6. Comparison of true and model impulse responses

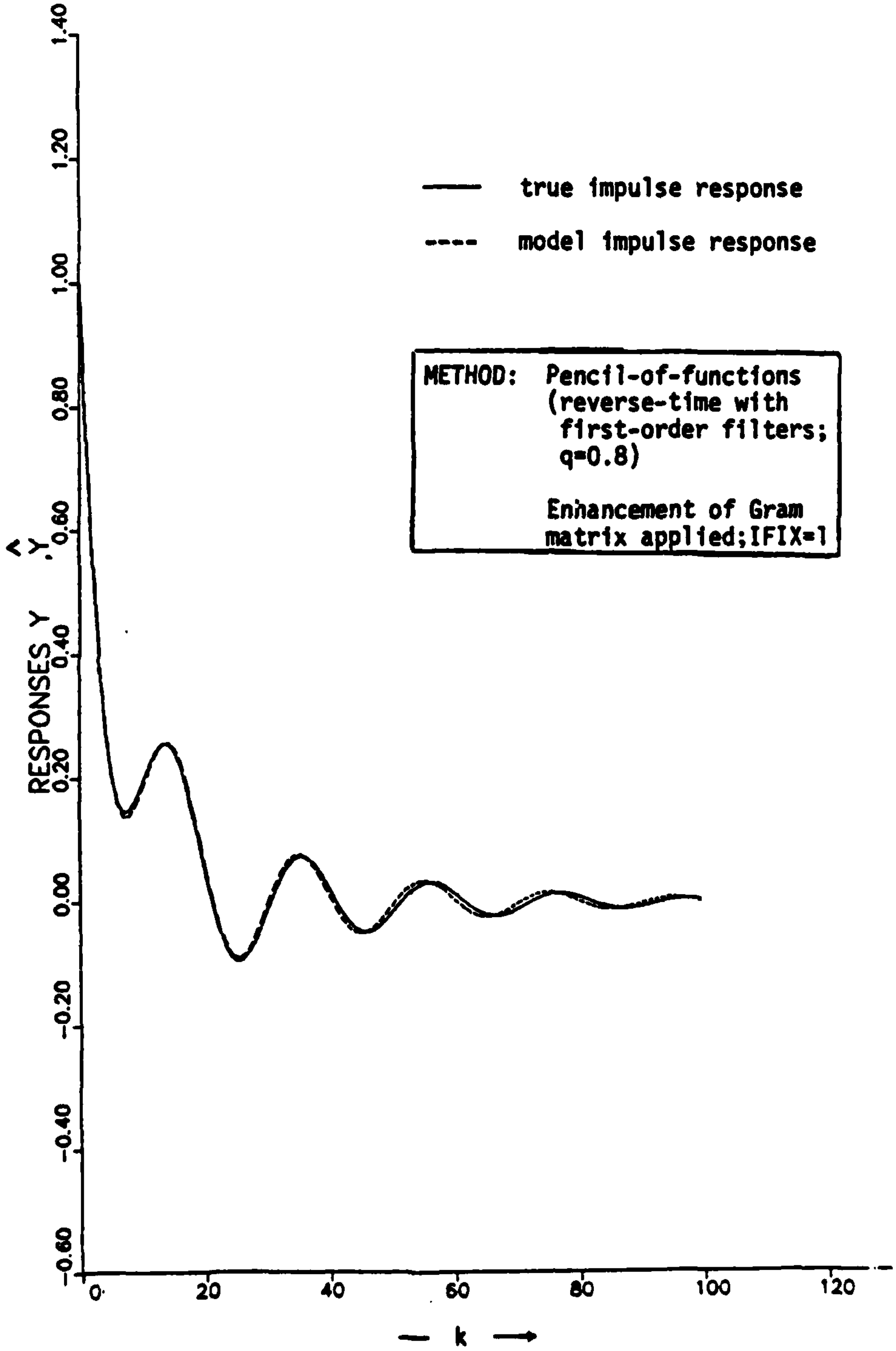


Fig. 7. Comparison of true and model impulse responses

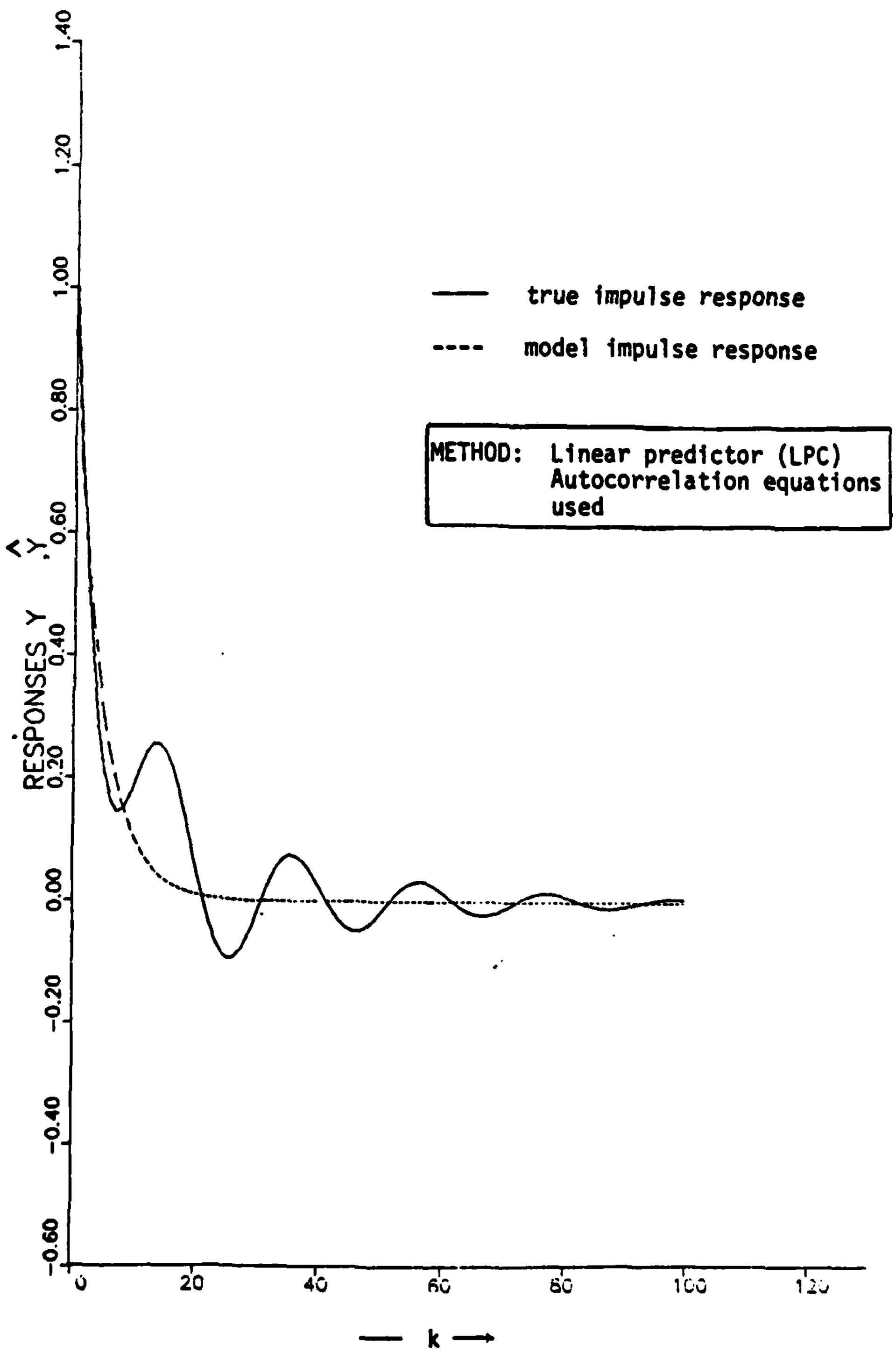


Fig. 8. Comparison of true and model impulse responses

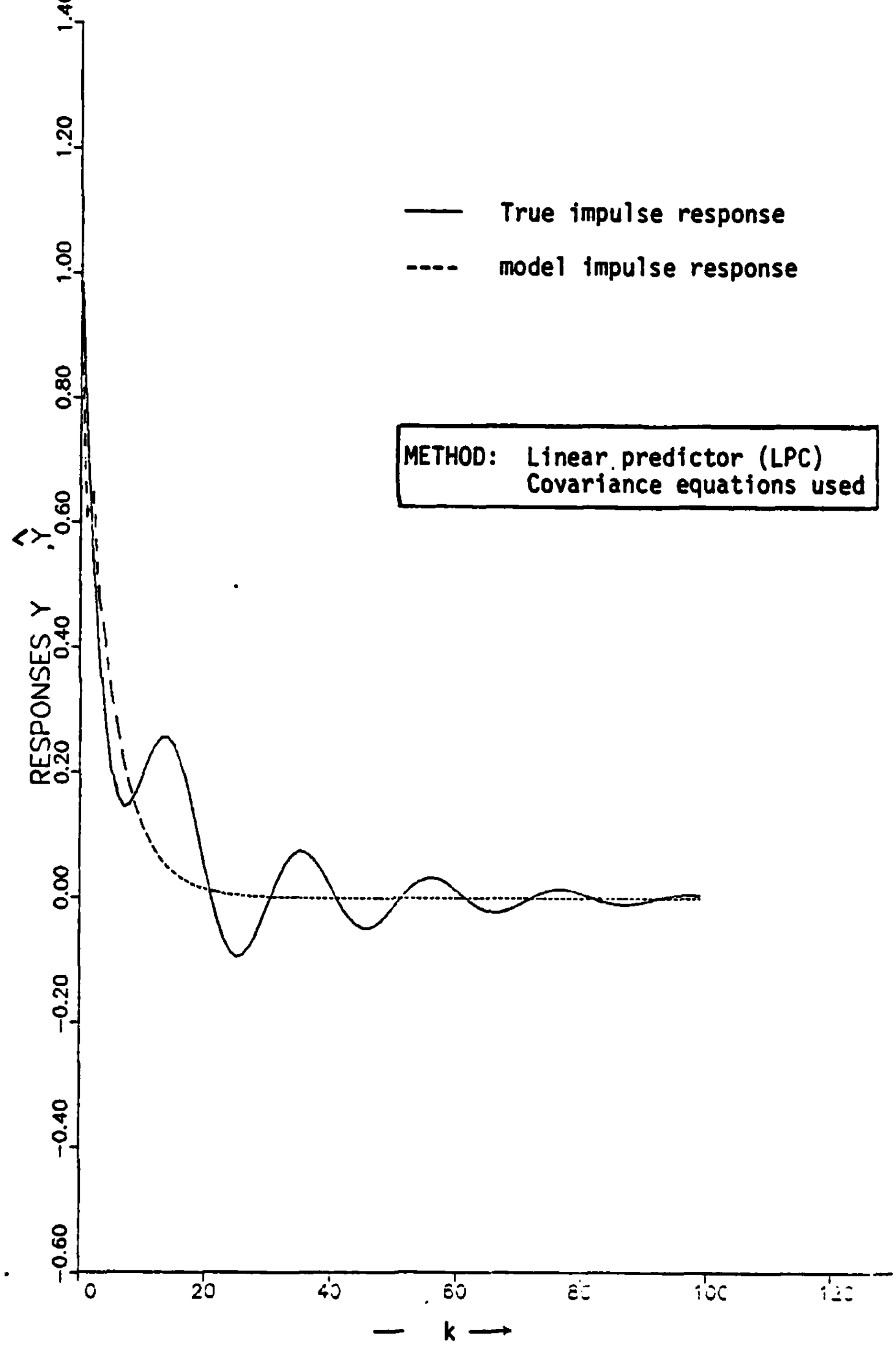


Fig. 9. Comparison of true and model impulse responses

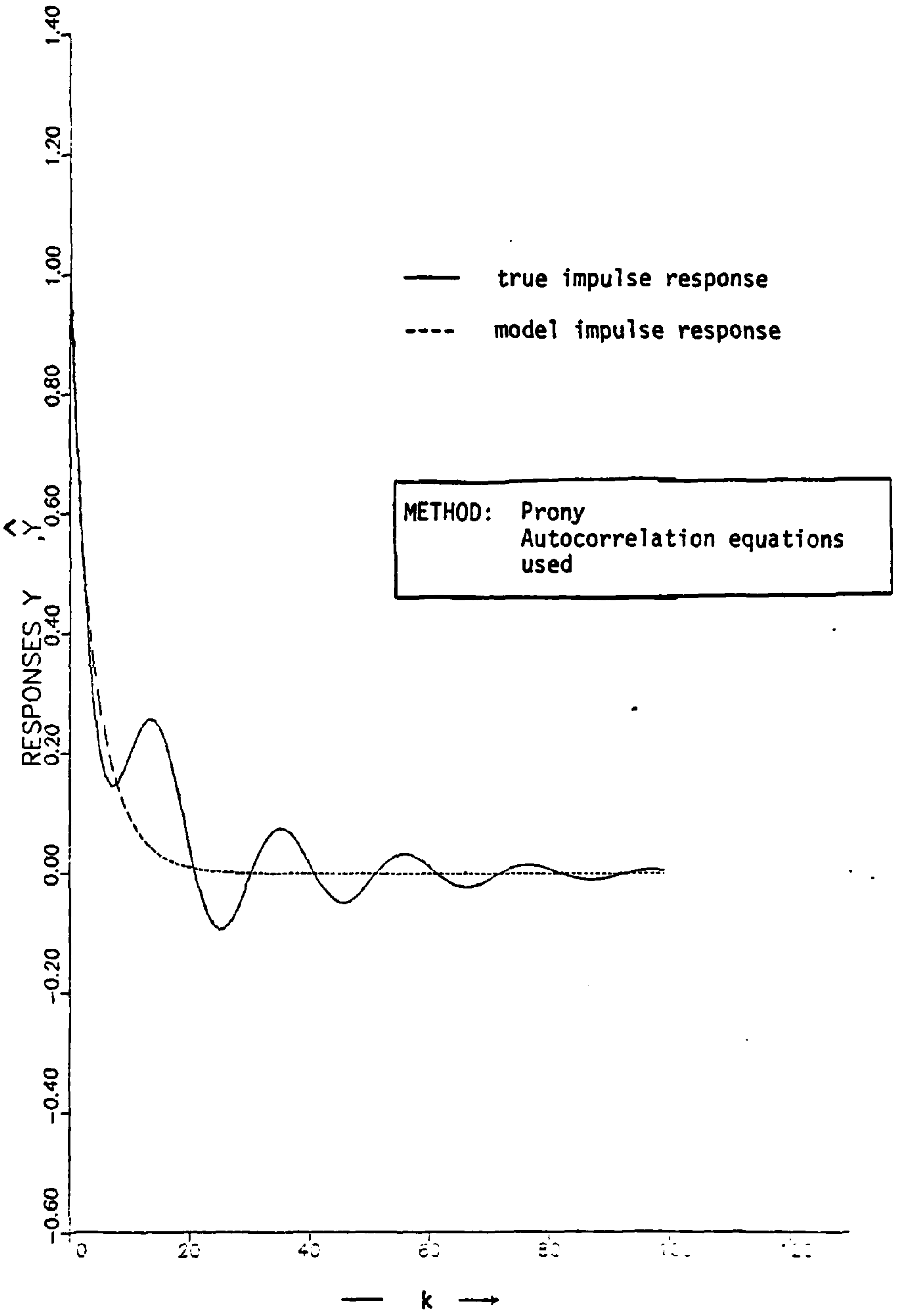


Fig. 10. Comparison of true and model impulse responses

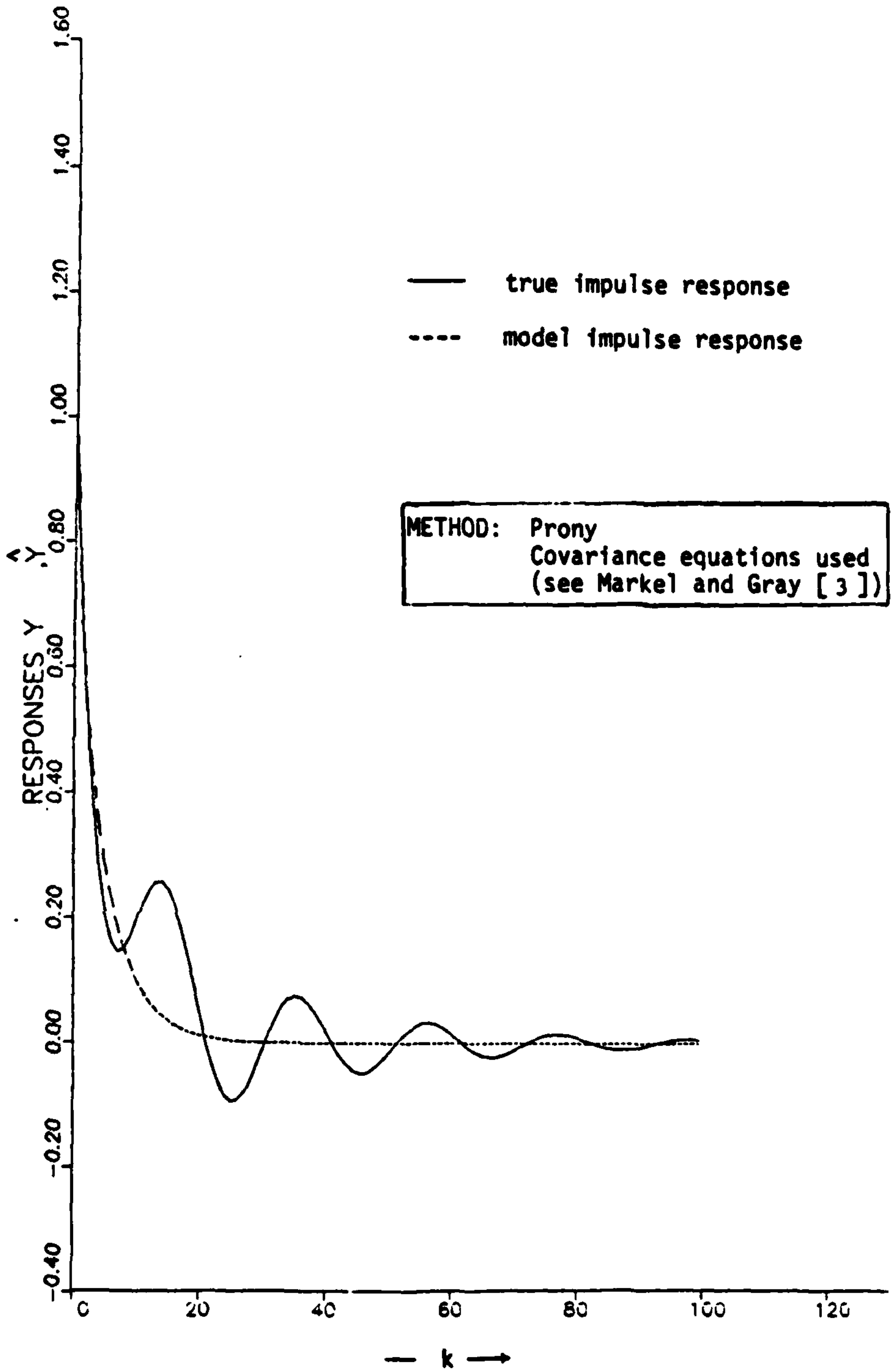


Fig. 11. Comparison of true and model impulse responses

This example demonstrates the superiority of the pencil-of-functions method over two other widely used methods (for modeling impulse responses), namely the all pole linear predictor and the prony method.

Example 2

As a real world application we consider the use of pencil-of-functions method to the transient response of a conducting pipe tested at the ATHAMAS-I EMP simulator. The conducting pipe is 10 m long and 1 m in diameter. Hence, the true resonance of the pipe is expected to be in the neighborhood of 14 MHz MHz. Also, the pipe has been excited in such a way that it is reasonable to expect only odd harmonics at the scattered fields. The data measured are the integral of the E-field; t.e., the measured variable is a voltage. The transient response used for analysis is shown in Fig. 12 by the solid line. The results of analysis by the pencil-of-functions method are given in Appendix C for the case of an 8th order model; the model response, with an error of 0.0125, is shown in Fig. 12 by the dotted line. Model poles are:

fundamental	$-4.280 \pm j\ 67.686$	Mrad/s	(=10.794 MHz)
3rd harmonic	$-22.470 \pm j218.200$	"	(=34.911 MHz)
curve-fit pair	$-2.543 \pm j\ 12.890$	"	(= 2.051 MHz)
curve-fit pair	$-16.547 \pm j\ 88.981$	"	(=14.404 MHz)

Note that a pole at the origin (due to the integrator) has not been obtained because we have used the bias extraction option. On the other hand, the curve-fit pole pair arises because the data do not truly pertain to the impulse response of a finite order (lumped) linear system.

Next the data were differentiated (actually differenced), and analyzed by the pencil-of-functions method. The results are given in Appendix C for 8th order analysis; the model response, with a fractional

energy error of 0.0369 is shown in Fig 13 by the dotted line (the solid line depicts the differentiated data). The model poles are:

fundamental	$-9.453 \pm j\,71.596$	Mrad/s	(=11.494 MHz)
3rd harmonic	$-25.996 \pm j222.340$	"	(=35.628 MHz)
5th harmonic	$-113.303 \pm j617.095$	"	(=99.855 MHz)
curve-fit pair	$-22.268 \pm j\,59.213$	"	(=10.068 MHz)

Here again a curve-fit pole pair arises because the data do not truly pertain to the impulse response of a finite order (lumped) linear system.

We also give below the poles arising from a 6th order model (the computer output is not given, nor the graphical display of the model response):

fundamental	$-9.575 \pm j\,75.106$	Mrad/s	(=12.050 MHz)
3rd harmonic	$-14.866 \pm j221.693$	"	(=35.363 MHz)
curve-fit pair	$-14.123 \pm j\,34.045$	"	(= 5.866 MHz)

The fractional energy error in modeling is 0.0566.

Here again a curve-fit pole pair arises because the data do not truly pertain to the impulse response of a finite order (lump ed) linear system.

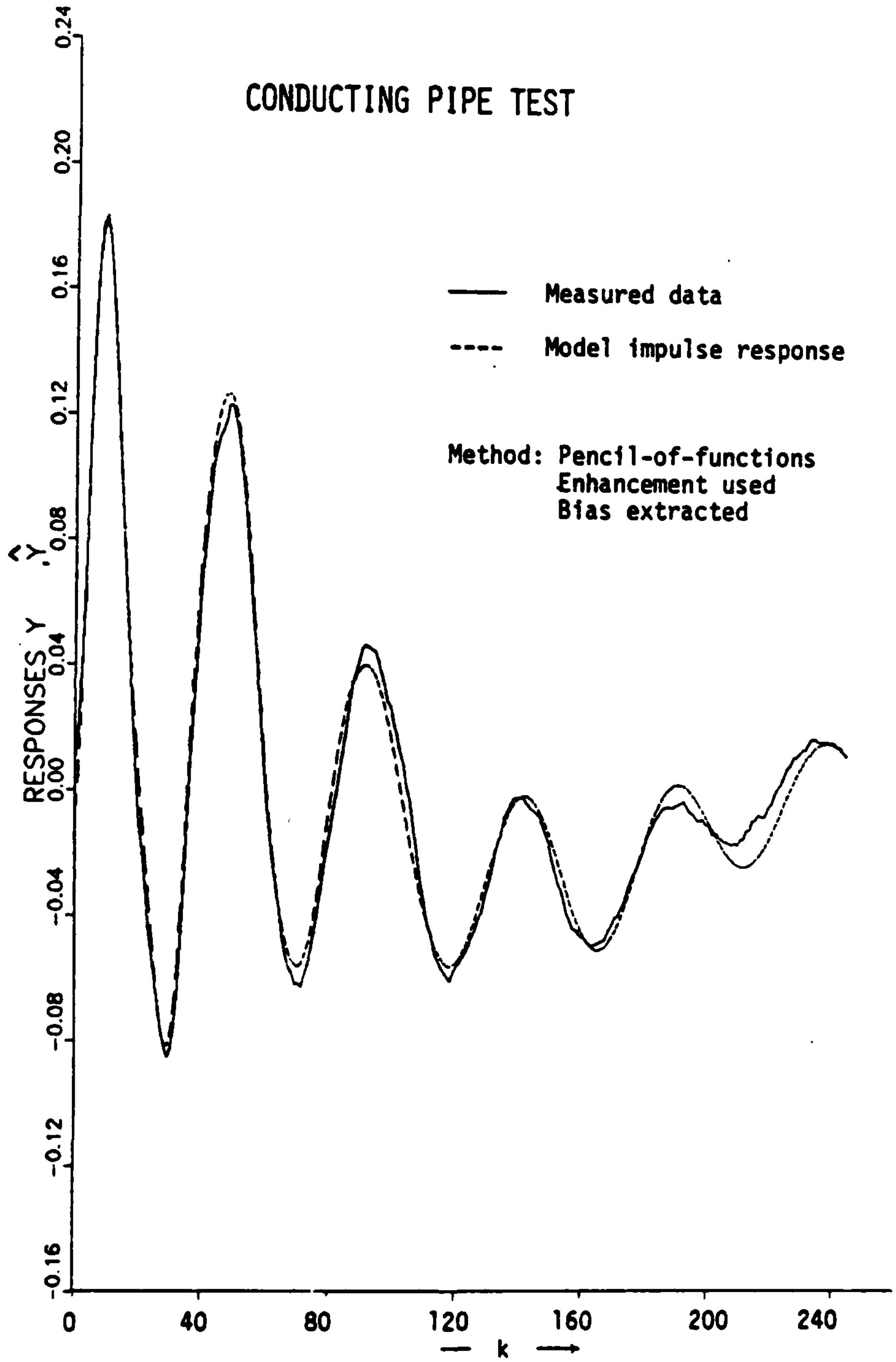


Fig. 12. Comparison of measured data (of the response of of a conducting pipe) and model impulse response

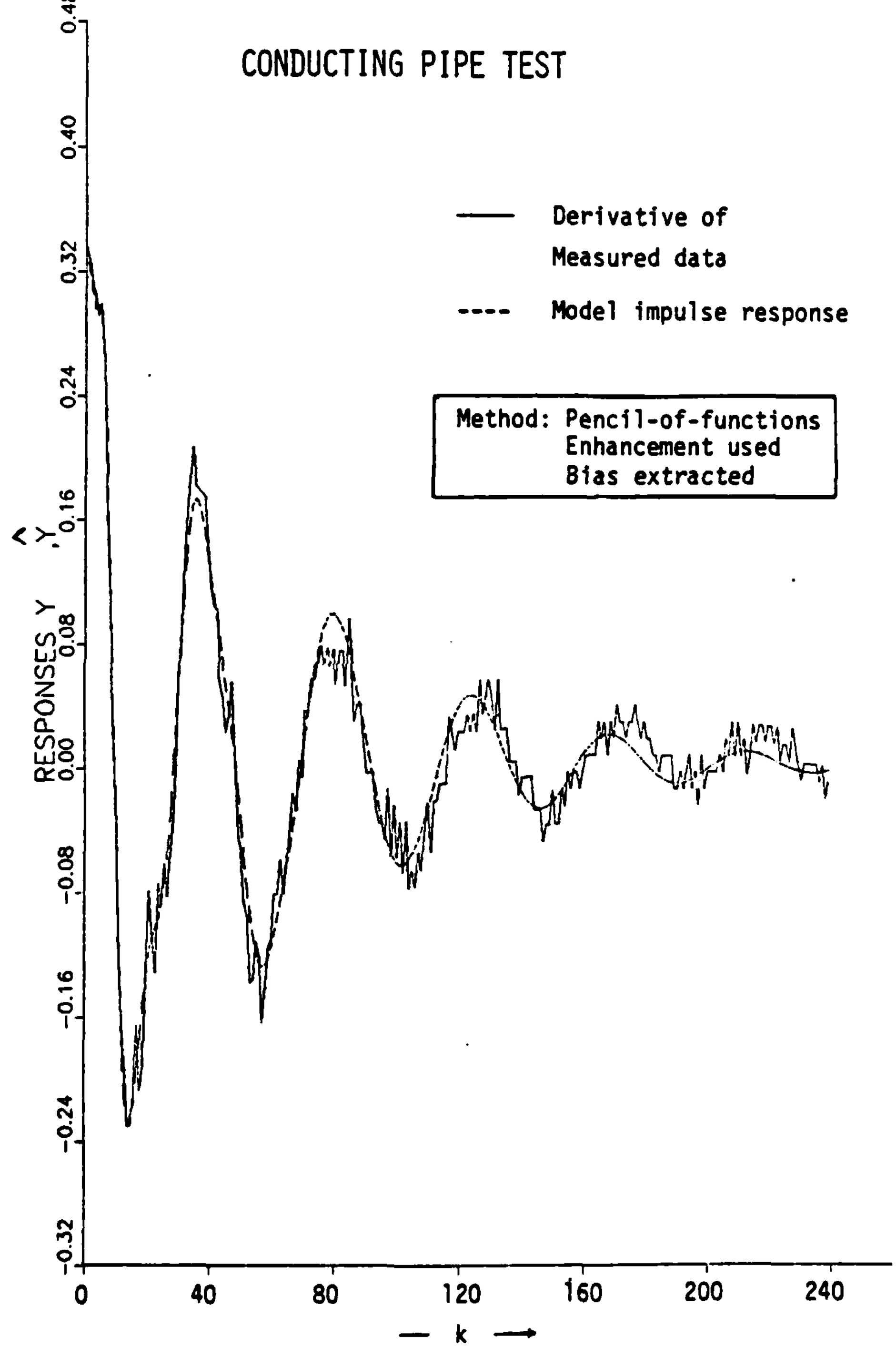


Fig. 13. Comparison of the derivative of measured data and the corresponding model impulse response

REFERENCES

[1] J. Makhoul, "Linear prediction: A tutorial review," IEEE Proc., Vol. 63, pp. 561-580, April 1980.

[2] T. K. Sarkar, V. K. Jain, J. Nebat, D. D. Weiner, "A comparison of the pencil-of-functions method with Prony's method Wiener filters and other identification techniques," Technical Rep. to Air Force Weapons Lab. (Contract F 33615-77-C-2059, Task 12), December 1977.

[3] J. D. Markel and A. H. Gray, Linear Prediction of Speech. Berlin: Springer-Verlag, 1976.

[4] V. K. Jain, "Filter analysis by use of pencil-of-functions: Part I," IEEE Trans. Circuits and Systems, Vol. CAS-21, pp. 574-579, September 1974.

[5] B. Gold, and C. M. Rader, Digital Processing of Signals. New York: McGraw-Hill, 1969.

[6] E. W. Cheney, Introduction to Approximation Theory. New York: McGraw-Hill, 1966.

[7] J. H. Carr, III, "An analytical investigation of transients synthesis by exponentials," M.S. Thesis, Mass. Inst. Tech., Cambridge, 1949.

[8] V. K. Jain, and R. D. Gupta, "Identification of linear systems through a Grammian technique," Intntl. J. Control, Vol. 12, pp. 421-431, 1970.

[9] V. K. Jain and J. S. Osman, "Computer routines for use in second-order Volterra identification of EMI," RADC-TR-79-266, October 1979.

[10] G. J. Bierman, Factorization Methods for Discrete Sequential Estimation. New York: Academic Press, 1977.

[11] M. L. Van Blaricum, "PSPL: A computer program for extracting the poles and residues from a transient signal using Prony's method," Scientific Report No. 76-2, Electromagnetics Laboratory, University of Illinois, Urbana, 1976.

APPENDIX A

DETAILS OF PROGRAM POF-FILTER

LSITINGS

PURPOSE, EQUATIONS

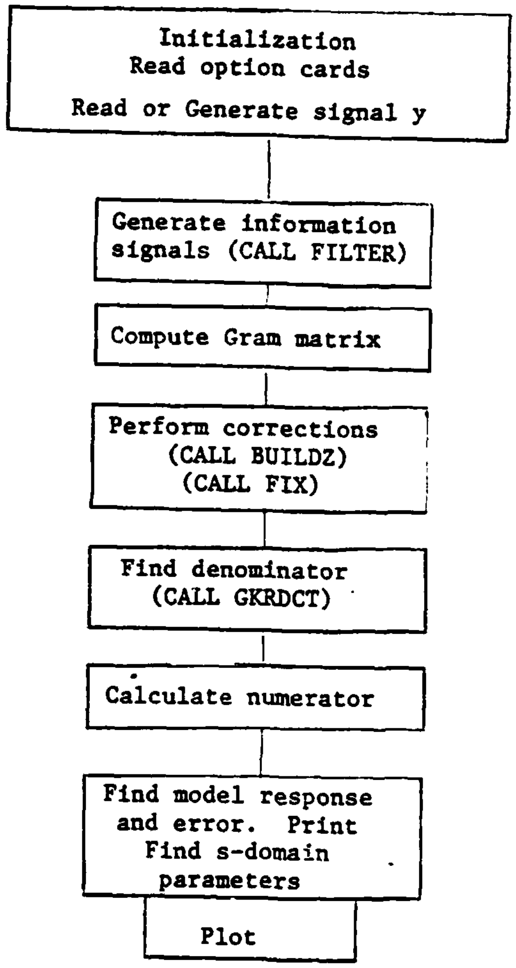
FLOWCHART

VARIABLES

FOR THE MAIN AND SUBROUTINES

MAIN PROGRAM

FLOWCHART



LISTING OF POF-FILTER

```
                    PROGRAM "POF-FILTER"
                    IMPULSE-RESPONSE MODELING
                    BY PENCIL-OF-FUNCTIONS METHOD
                    APRIL 1980
                    * DECOUPLED DENOM. AND NUM. DETERMINATION*
                    * FIRST-ORDER FILTERS USED*
                    * NOISE CORRECTION OPTION*
                    * BIAS CORRECTION OPTION*
                    * DIRECT TRANSMISSION OPTION*
                    * RESULTS IN BOTH Z- AND S- DOMAINS*

    "POF-FILTER"    MODELS IMPULSE RESPONSE OF A SCATTERER/CHANNEL/NETWORK
                    IT CAN BE USED IN SIMULATION MODE
                    OR ON EXPERIMENTALLY RECORDED RESPONSES.
                    FOR COMPARISON PURPOSES IT ALSO PROVIDES
                    ON AN OPTIONAL BASIS THE FOLLOWING METHODS
                    # LINEAR PREDICTOR (COV OR AUTOCORR)
                    # PRONY
                    # LEAST-SQUARES PRONY


      NOTE: 1350 LINES OF CODE. THIS CAN BE REDUCED SUBTANTIALLY
      FOR PARTICULAR APPLICATIONS. IN PARTICULAR, ROUTINES "PLOP"(47),
      "ZTOS"(157), AND "POLRT"(201) MAY BE ELIMINATED
      IF ONLY Z-DOMAIN RESULTS NEEDED
      ******************************
      DIMENSION F(2500),U(800),UU(800),X(800,11),G(11,11),AM(11,11)
      DIMENSION GN(11,11),GEST(11,11),GDUM(11,11),E(11,11),EN(11,11)
      DIMENSION V(22),VV(22),AMF(11),SR(11),SI(11),SPH(11)
      DIMENSION TITLE(70),IBUF(512),IDUM(10)
      DOUBLE PRECISION DT,AC,BD,ERROR
      COMMON /DA0/ISPN,DELTA,SIG2,DT,QI,BIAS,IBIAS,DFAC
      COMMON /DA1/FBAR,EBAR,FESUM,EESUM
      COMMON /IO/IR,ILT,IPR,ITPR,IZPR,IROUND,IPLT
      REWIND5
      MAXPL=800
      MAX=11
      MAX2=2*MAX
      IR=5
      ILT=6
      ISKIP=0
      CALL VEQUAT(MAXPL,U,F,0,10)
      CALL VEQUAT(MAXPL,UU,F,0,11)
      CALL VEQUAT(MAX2,V,VV,0,0)
      WRITE(ILT,2)
      READ(IR,8)(TITLE(I),I=1,70)
      WRITE(ILT,18)(TITLE(I),I=1,70)
      READ(IR,8)(TITLE(I),I=1,70)
      READ(IR,8)(TITLE(I),I=1,70)
      READ(IR,4)NPT,IXX,IYY,N,ISIM,NCOMP,IPLT,NNPT,
     +YYYY,DT,BIAS,ANBIAS,VAR
      NP1=N+1
      NP2=NP1+1
      NP3=N+3
      NPNP2=N+N+2
      NPNP1=N+N+1
      IF(NNPT.EQ.0)NNPT=NPT
      IF(DT.EQ.0.0)DT=1.0
      IFUN=2
```

```
      IF(IPLT.LT.0) IFUN=1
      IF(IPLT.LT.0) IPLT=-IPLT
      FMAX=1.0
      IF(ISIM.EQ.3) GO TO 63
      IF(ISIM.NE.-1)GO TO 59
      IMAX=0
      DO 58 I=1,NNPT,10
      READ(IR,161)(IDUM(J),J=1,10)
      DO 53 J=1,10
      IF(IABS(IDUM(J)).GT.IMAX)IMAX=IABS(IDUM(J))
      K=I-1+J
3     F(K)=IDUM(J)
8     CONTINUE
      FMAX=IMAX
      GO TO 45
9     CONTINUE
      IF(ISIM.NE.0) GO TO 49
      READ(IR,163)(F(K),K=1,NNPT)
      FMAX=0.0
      DO 41 K=1,NPT
      IF(ABS(F(K)).GT.FMAX)FMAX=ABS(F(K))
1     CONTINUE
5     NBIAS=ANBIAS
      IF(NBIAS.EQ.0)NBIAS=0.2*NNPT
      NDIE=NNPT+1-NBIAS
      FB=0.0
      DO 57 K=NDIE,NNPT
      FB=FB+F(K)
      FB=FB/NBIAS
      DO 56 K=1,NNPT
      F(K)=(F(K)-FB)/FMAX
      GO TO 61
      CONTINUE
      IF(ISIM.EQ.1) READ(IR,160) (V(I),I=1,NP1)
      IF(ISIM.EQ.1) READ(IR,160) (V(I),I=NP2,NPNP2)
      CALL VEQUAT(NP1,V(NP2),-1.0,0.3)
      IF(ISIM.EQ.1) CALL RESPON(F,L,N,V,VV,NNPT)
      IF(ISIM.EQ.1) GO TO 61
      DO 60 I=1,NCOMP
      READ(IR,5) AMP(I),SR(I),SI(I),SPH(I)
      WRITE(ILT,11) I,AMP(I),SR(I),SI(I),SPH(I)
      CALL SIGNAL(F,NNPT,AMP,SR,SI,SPH,DT,NCOMP)
      CONTINUE
      IF(IPLT.GE.1) CALL PLOTS(IBUF,512,9)
 11   READ(IR,8) (TITLE(I),I=1,70)
      IF(EOF(IR).NE.0)GO TO 998
      WRITE(ILT,3)
      WRITE(ILT,18) (TITLE(I),I=1,70)
      READ(IR,8) (TITLE(I),I=1,70)
      WRITE(ILT,18) (TITLE(I),I=1,70)
      BACKSPACE 5
      READ(IR,6) IPR,IZTS,IREM,ISPN,IFIX,NFIX,IBIAS,IB0,MNPT,QI,DFAC
      NSTRT=2
      IF(ISPN.EQ.-2)NSTRT=NP1
      IF(ISPN.EQ.-3)NSTRT=1
      IZPR=0
      IF(IPR.GE.20.AND.IPR.LE.39)IZPR=(IPR-10)/10
      ITPR=0
      IF(IPR.GE.90) ITPR=1
      IPR=IPR-10*(IPR/10)
```

```
      IF(ISPN.LE.-2.AND.IREM.GT.N)IREM=N
      IF(DFAC.EQ.0)DFAC=0.1
      IBS=1
      IF(IBIAS.EQ.0)IBS=0
      IF(IBIAS.LE.0)IBIAS=0
      IROUND=0


      CORRUPT SIGNAL IF DESIRED.
      PROCESS WITH FIRST ORDER FILTERS

10    CONTINUE
      IF(VAR.GE.1.0E-6)CALL CORUPT(F,X,VAR,NPT,MAXPL)
5     CONTINUE
      IF(VAR.GE.1.0E-6)GO TO 99
      DO 30 K=1,NPT
]     X(K,1)=F(K)+BIAS
9     CONTINUE
      INT=-1
      IF(ISPN.LE.-2)INT=2
      IF(NP1.GT.1)CALL FILTER(X,NPT,NP1,MAXPL,INT)

      COMPUTE GRAM MATRIX

      NPP=NP1
      IF(IBIAS.NE.0)NPP=NP2
      DO 44 I=1,NPP
      DO 44 J=1,NPP
      AD=0.0
      IF(ISPN.EQ.0.AND.IROUND.EQ.0) GO TO 43
      DO 42 K=NSTRT,NPT
      AD=AD+X(K,I)*X(K,J)
      GN(I,J)=AD*DT
      IJ1=IABS(I-J)+1
      IF(ISPN.EQ.-3.AND.I.GE.2)GN(I,J)=GN(1,IJ1)
      GDUM(I,J)=GN(I,J)
      CONTINUE
      IF(IROUND.EQ.0)G(I,J)=GN(I,J)
      CONTINUE
      IF(ISPN.NE.0.OR.IROUND.NE.0)
     1CALL GKROCT(GN,E,DET,V,NPP,NPP,MAX,1)
      IF(IROUND.EQ.0)WRITE(ILT,171)DET
      IF(IROUND.EQ.1)WRITE(ILT,172)DET
      IF(IPR.GE.1)CALL PRTMAT(GN,NPP,NPP,MAX,-1)
      WRITE(ILT,1)
      IRO=IROUND
      IF(IROUND.EQ.0)IROUND=IROUND+1
      IF(IRO.EQ.0.AND.ISPN.GT.-1)GO TO 410
      IF(IFIX.EQ.-1)GO TO 203


      ESTIMATE OF  **  G

6     CALL BUILDZ(AM,V,NP1,NPT,MAX,NFIX)
      -----NP1 REPLACED BY NPP NEXT 3 CARDS--
      CALL FIX(GDUM,AM,GEST,E,V,NPP,NPP,SIG2,MAX,IFIX)
      IF(IFIX.EQ.1)WRITE(ILT,462)SIG2
      CALL GKROCT(GEST,E,DET,V,NPP,NPP,MAX,1)
      WRITE(ILT,162)DET
      IF(IPR.GE.1)CALL PRTMAT(GEST,NP1,NP1,MAX,0)
      DO 154 I=1,NP1
      DO 154 J=1,NP1
      GDUM(I,J)=GEST(I,J)
      NFIX=NFIX-1
```

```
      IF(NFIX.GE.1)GO TO 156
      ISKIP=1
      IROUND=0



      DETERMINE NUMERATCR

 13   CONTINUE
      IF(IBS.EQ.1)IBIAS=1
      IF(ISPN.EQ.0)GO TC 998
      CALL VEQUAT(NP1,V(NP2),VV,0,0)
      V(NP2)=-1.0
      CALL RESPON(X(1,1),U,N,V,VV,NPT)
      IF(IPR.GE.4)WRITE(ILT,174)
      IF(IPR.GE.4)WRITE(ILT,210)(X(K,1),K=1,NPT)
      CALL FILTER(X,NPT,NP1-IREM,MAXPL,2)
      L=N-IREM
      IF(IBIAS.NE.0)L=L+1
      LP1=L+1
      LP2=L+2
      IF(IBIAS.NE.0)CALL VEQUAT(NPT,X(1,LP1),U,0,11)
      CALL VEQUAT(NPT,X(1,LP2),F,0,1)
      CALL VEQUAT(NPT,X(1,LP2),BIAS,0,4)
      IF(IPR.GE.5)WRITE(ILT,110)((X(K,I),K=1,NPT),I=1,LP2)
      L=L+IB0
      LP1=L+1
      DO 216 I=1,L
      DO 216 J=1,LP1
      AD=0.0
      DO 215 K=1,NPT
 5    AD=AD+X(K,I+1-IB0)*X(K,J+1-IB0)
 6    G(I,J)=AD*DT
 5    IF(IPR.GE.5)CALL PRTMAT(G,L,LP1,MAX,205)
      CALL GKROOT(G,E,DET,VV,L,L,MAX,0)
 7    IF(IPR.GE.5)CALL PRTMAT(E,L,L,MAX,207)
      CALL VEQUAT(NP1,VV,AMP,0,0)
      DO 220 I=1,L
      AD=0.0
      DO 219 J=1,L
 9    AD=AD+E(I,J)*G(J,LP1)
 0    VV(I)=AD
      FMEAN=0.0
      IF(IBIAS.NE.0)FMEAN=VV(L)
      CALL VEQUAT(IREM,VV(N+IB0-IREM+1),AMP,0,0)
      V(NP2)=0.0
      CALL VEQUAT(N+IB0,V(NP3-IB0),VV,0,1)
      IF(IPR.GE.2)WRITE(ILT,12)FMAX
 1    WRITE(ILT,303)FMAX
      WRITE(ILT,210)(V(I),I=1,NP1)
      WRITE(ILT,210)(V(I),I=NP2,NPNP2)
      IF(IBIAS.NE.0)WRITE(ILT,305)FMEAN

      MODEL RESPONSE, AND ERROR.

      IF(MNPT.EQ.0) MNPT=NNPT
      CALL VEQUAT(NP1,V(NP2),-1.0,0,3)
      CALL RESPON(X(1,2),U,N,V,VV,MNPT)
      CALL VEQUAT(MNPT,X(1,2),FMEAN,0,4)
      ERROR=0.0
      FFSUM=0.0
      DO 213 K=1,MNPT
      X(K,1)=F(K)*FMAX+BIAS
```

```
      X(K,2)=X(K,2)*FMAX
      FFSUM=FFSUM+X(K,1)*X(K,1)
      X(K,3)=X(K,1)-X(K,2)
 13   ERROR=ERROR+X(K,3)*X(K,3)
      FFSUM=FFSUM*DT
      ERROR=ERROR*DT
      RATIO=ERROR/FFSUM
      WRITE(ILT,304)ERROR,FFSUM,RATIO
      IF(IPR.GE.2)WRITE(ILT,112)
      IF(IPR.GE.2)WRITE(ILT,110)(X(K,1),K=1,MNPT)
      IF(IPR.GE.2)WRITE(ILT,113)
      IF(IPR.GE.2)WRITE(ILT,110)(X(K,2),K=1,MNPT)
      IF(IPR.GE.2)WRITE(ILT,114)
      IF(IPR.GE.2)WRITE(ILT,110)(X(K,3),K=1,MNPT)
      DELT=DT
      IF(IZTS.GE.0)CALL ZTOS(V(1),V(NP2),N,DELT,IZTS)

      IF(IPLT.EQ.0)GO TC 239
      IF(IPLT.EQ.1)GO TO 238
      DO 230 I=1,2
      KK=0
      DO 230 K=1,MNPT,IPLT
      KK=KK+1
 :0   X(KK,I)=X(K,I)
      MNPT=MNPT/IPLT
 :8   T0=0.0
      DELT=DT*IPLT
      CALL PLOP(MNPT,IFUN,X,MAXFL,T0,DELT,1HY,1HT,IBUF)
 :9   CONTINUE
      FORMAT  STATEMENTS

      FORMAT(I4,6I2.I4,6F10.0)
      FORMAT(5F10.3)
      FORMAT(8I2,I4,4F10.0)
      FORMAT(70A1)
      FORMAT(2X,70A1)
  3   FORMAT(10(5X,F5.0))
  4   FORMAT(10(5X,I5))
      FORMAT(2X,I2,* AMP=*,F8.2,*  S=*,F10.4,*  + J*,F10.4,
     1*  PHASE=*,F10.4)
      FORMAT(2X,*WAVEFORMS AND NUMER. SCALED BY XMAX=*,F12.5)
  0   FORMAT(10X,8HG MATRIX)
  0   FORMAT(10X,8HM MATRIX)
  2   FORMAT(10X,11HGEST MATRIX,*  (DET=*,G13.6,*)*)
  0   FORMAT(2X,5(2X,G13.6))
  10  FORMAT(20(1X,F5.2))
  0   FORMAT(2X,10(F7.4))
  2   FORMAT(2X,*ORIGINAL SIGNAL(INCLUDES BIAS,IF ANY)*,/)
  3   FORMAT(2X,*IMPL.RESP OF MODEL(INC.3-HAT,IF IBIAS.NE.0)*,/)
  4   FORMAT(2X,*ERROR=F(K)-FREC(K)*,/)
  8   FORMAT(10X,14HNOISY X MATRIX)
  9   FORMAT(10X,8HX MATRIX)
  1   FORMAT(10X,16HTRUE GRAM MATRIX,* (DET=*,G13.6,*)*)
  2   FORMAT(10X,17HNOISY GRAM MATRIX,* (DET=*,G13.6,*)*)
  4   FORMAT(2X,*IMPULSE RESPONSE OF 1/A(Z)*)
  0   FORMAT(5F10.0)
  1   FORMAT(10I5)
  3   FORMAT(10F8.6)
  3   FORMAT(2X, *EST TF B(Z)/A(Z)*,*  (FMAX=*,E12.5,*)*)
  4   FORMAT(/,2X,*SS ERROR=*,G13.6,*SS SIGNAL=*,G13.6,
     +*RATIO=*,G13.6,/)
  5   FORMAT(2X,*ESTIMATED MEAN=*,G13.5)
  2   FORMAT(*  ESTIMATED NOISE VAR=*,G12.5)
      FORMAT(/)
```

```
      FORMAT(1H1)
      FORMAT(/////)

      GO TO 1111
 98   CONTINUE
      CALL PLOT(0.,0.,999)
      STOP
      END
```

SUBROUTINE: BUILDR

PURPOSE: To generate a conversion matrix to go from $\sqrt{D_i}$ to parameters a_i of $A(z)$. See equation (9) of Section I

EQUATIONS:

Conversion matrix (shown below for n=3)

$$= \text{Diag}\{q^3, q^2, q, 1\} \begin{bmatrix} 1 & & & \\ -3 & 1 & & \\ 3 & -2 & 1 & \\ -1 & 1 & -1 & 1 \end{bmatrix}$$

ROUTINE VARIABLES

A Conversion matrix (not to be confused with the denominator polynomial)

X Vector which brings in $[\sqrt{D_1} \ \sqrt{D_2} \ \ldots \ \sqrt{D_N}]$ to the routine and takes back the denominator parameters $[a_0 \ a_1 \ \ldots \ a_n]$

N System order plus one

MAX Maximum dimensionality of matrix A

FURTHER DESCRIPTION:

```
      SUBROUTINE BUILDR(A,X,N,MAX)
C     ---------------------------------------
C     CONVERSION MATRIX: REVERSE FOF PROCESSING -- I.R. MODELING
      DIMENSION A(MAX,1),X(1),Y(20)
      DOUBLE PRECISION DT,Y
      COMMON /DAO/ISPN,DELTA,SIG2,DT,QI,BIAS,ISIAS,DFAC
      COMMON /IO/IR,ILT,IPR,ITPR,IZPR,IROUND,IPLT
      NM1=N-1
      DO 11 I=1,N
      Y(I)=0.0
      DO 11 J=1,N
      A(I,J)=0.0
      A(N,N)=1.0

      DO 20 JJ=1,NM1
      J=N-JJ
      DO 15 KK=1,2
      K=KK-1
      DO 15 I=J,NM1
      A(I+K,J)=A(I+K,J)+A(I+1,J+1)*(1.0-K-K)
      CONTINUE
      QQ=1.0
C     CHANGED THRU 22 3-17-80
      DO 22 II=1,NM1
      I=N-II
      QQ=QQ*QI
      DO 22 J=1,N
      A(I,J)=QQ*A(I,J)

      DO 25 I=1,N
      IF(IPR.GE.3)WRITE(ILT,5)(A(I,JJ),JJ=1,N)
      DO 25 J=1,N
      Y(I)=Y(I)+A(I,J)*X(J)
      DO 28 I=1,N
      X(I)=Y(I)/Y(1)


      IF(IPR.GE.3)WRITE(6,7)(X(I),I=1,N)
      FORMAT(2X,10G12.5)
      FORMAT(*  ESTIMATED PARAMETER VECTOR*,/,10G13.6)
      RETURN
      END
```

SUBROUTINE: BUILDZ

PURPOSE: To calculate unit noise covariance matrix for reverse-time first-order filtering case (under further development)

EQUATIONS:

$$Z = \sum_{k=1}^{K} (K-k+1)\, \underline{m}(k)\, \underline{m}^T(k)$$

where $\underline{m}(k)$ is the vector of unit-pulse responses of the connection filters

ROUTINE VARIABLES

Z	Covariance matrix for unit noise
R	Work vector
NP1	system order plus one
NDIM	Maximum dimension of the matrix Z
NPT	Number of points in signal
NFIX	Not used (blank)

FURTHER DESCRIPTION:

```
      SUBROUTINE BUILDZ(Z,R,NP1,NPT,NDIM,NFIX)
      -----------------------------------------
      ALTERNATIVE NOISE COV PGM FOR *GNN*
      DIMENSION Z(NDIM,1),R(1)
      DOUBLE PRECISION DT
      COMMON /DA0/ISPN,DELTA,SIG2,DT,QI,BIAS,IBIAS,DFAC
      COMMON /IO/IR,ILT,IPR,ITPR,IZPR,IROUND,IPLT
      Q=QI
      TIME=DT*NPT
      IOPT=NFIX+1
      GO TO(201,101,101,201),IOPT
 11   SC=DT
      N=NP1-1
      R(1)=1.0
      DO 1 I=1,NP1
      IF(I.GE.2)R(I)=R(I-1)
      DO 1 J=1,NP1
      Z(I,J)=0.0
      DO 2 K=1,NPT
      NPTK=NPT+1-K
      DO 3 J=1,NP1
      DO 3 I=J,NP1
      Z(I,J)=Z(I,J) +R(I)*R(J)*NPTK
      R(1)=0.0
      DO 4 I=1,N
      R(I+1)=Q*R(I+1)+R(I)
      CONTINUE
      DO 7 J=1,NP1
      DO 7 I=J,NP1
      Z(I,J)=Z(I,J)*DT**(I+J-1)
      GO TO 301
  1   CONTINUE
      DO 210 J=1,NP1
      DO 210 I=J,NP1
      IF(I.EQ.1)Z(1,1)=TIME
  0   Z(I,J)=(TIME**(I+J-1))/(I+J-1)
      WRITE(ILT,161)
  1   CONTINUE
      DO 174 I=1,NP1
      DO 168 J=I,NP1
  5   Z(I,J)=Z(J,I)
      IF(IPR.GE.2)WRITE(ILT,220)(Z(I,J),J=1,NP1)
  4   CONTINUE
  3   FORMAT(10X,* QUANT. NOISE *)
  1   FORMAT(10X,*BIAS EFFECT*)
  3   FORMAT(2X,5(2X,G13.6))
      RETURN
      END
```

SUBROUTINE: CORUPT

PURPOSE: This routine, useful in simulation mode, can be called when it is desired to add random noise of given variance to the signal

EQUATIONS:

ROUTINE VARIABLES

F	Input signal
X	First column of X would contain the corrupted signal; the second column temporarily contains the noise added to signal
VAR	Variance of noise added to signal
NPT	Number of signal points
NDIM	Maximum column dimensionality of X

FURTHER DESCRIPTION:

This routine needs a library routine to produce random numbers (gaussian, zero mean and uncorrelated)

```
      SUBROUTINE CORUPT(F,X,VAR,NPT,NDIM)
      -------------------------------------------------
      ADDS NOISE
      DIMENSION F(1),X(NDIM,1)
      DOUBLE PRECISION DT,AD,BD
      COMMON /DA0/ISPN,DELTA,SIG2,DT,QI,BIAS,IBIAS,DFAC
      COMMON /DA1/FBAR,EBAR,FESUM,EESUM
      COMMON /IO/IR,ILT,IPR,ITPR,IZPR,IROUND,IPLT
      FBAR=0.
      EBAR=0.
      FESUM=0.
      EESUM=0.
      WRITE(ILT,469)VAR
      IS=2458169
      IS2=397665
      SIGMA=SQRT(VAR)
      CALL NRML(NPT,1,1,0.,SIGMA,IS,IS2,X(1,2),0)
      DO 26 K=1,NPT
5     X(K,1)=F(K)+BIAS+X(K,2)

      DO 211 K=1,NPT
      FB=F(K)+BIAS
      FBAR=FBAR+FB
      EBAR=EBAR+X(K,2)
      EESUM=EESUM+X(K,2)*X(K,2)
      FESUM=FESUM+FB*X(K,2)
      IF(ISPN.EQ.0)X(K,1)=X(K,2)


1     CONTINUE
      EESUM=EESUM*DT
      FESUM=2.0*FESUM*DT
      FBAR=FBAR/NPT
      EBAR=EBAR/NPT
      WRITE(ILT,462)FBAR,EBAR,FESUM,EESUM
      IF(IPR.LE.2) GO TO 411
      WRITE(ILT,8)
      WRITE(ILT,110)(X(K,1),K=1,NPT)
      IF(ISPN.EQ.0)GO TO 411
      WRITE(ILT,18)
      WRITE(ILT,115)(X(K,2),K=1,NPT)
      WRITE(ILT,1)
1     CONTINUE
9     CONTINUE
      FORMAT STATEMENTS

      FORMAT(10X,16HROUNDED F SIGNAL)
      FORMAT(10X,16HROUNDOFF ERROR E)
]     FORMAT(2X,5(2X,G11.4))
]     FORMAT(20(1X,F5.2))
5     FORMAT(1X,20(1X,F5.3))
2     FORMAT(2X,5HFBAR=,E11.4,6H EBAR=,E11.4,5H FE2=,E11.4,4H EE=,E11.4)
9     FORMAT(2X,*VARIANCE OF NOISE=*,E12.4)
      FORMAT(/)
      RETURN
      END
```

SUBROUTINE: FILTER

PURPOSE: To produce the information signals. Specifically, this routine performs reverse-time first order filtering upon the given signal (stored in the first column of the matrix X)

EQUATIONS:

$$X(k,i+1) = q\ X(k+1,i+1) + X(k,i)$$

ROUTINE VARIABLES

X	Matrix of information signals. First column brings in the signal to be processed
NPT	Number of signal points
NP1	Number of information signals (model order plus one)
NDIM	Maximum column dimensionality of X
INT	Option parameter. -1 for reverse-time filtering, +2 for unit-shifts (or delays)

FURTHER DESCRIPTION:

```
      SUBROUTINE FILTER(X,NPT,NP1,NDIM,INT)
      -------------------------------------------------
      DIMENSION X(NDIM,1)
      DOUBLE PRECISION DT,SC,BD
      COMMON /DAO/ISPN,DELTA,SIG2,DT,QI,BIAS,IBIAS,DFAC
      COMMON /IO/IR,ILT,IPR,ITPR,IZPR,IROUND,IPLT
      GENERATE FIRST-ORDER FITER PROCESSED SIGNALS FROM DATA IN X(K,1)
      INT=1 OR 0 FOR FORWARD, -1 FOR REVERSE TIME
                 FIRST-ORDER FILTERING
      INT=2 FOR UNIT DELAYS (X(K,I+1)=X(K-1,I))
      N=NP1-1
      NP2=NP1+1
      IOPT=INT+2
      GO TO(51,11,11,91),IOPT
      FORWARD FIRST-ORDER FILTERING
 1    CONTINUE
      DO 40 J=1,N
      JJ=J+1
      X(1,JJ)=X(1,1)
      DO 40 K=2,NPT
      K1=K-1
      X(K,JJ)=QI*X(K1,JJ)+X(K,J)
 ]    CONTINUE
      GO TO 70
      REVERSE-TIME FIRST-ORDER FILTERING
      CONTINUE
      DO 60 J=1,N
      JJ=J+1
      X(NPT,JJ)=X(NPT,1)
      X(NPT,JJ)=0.0
      BD=X(NPT,JJ)
      DO 60 KK=2,NPT
      K=NPT+1-KK
      K1=K+1
      CHANGED NEXT CARD 3/17/80
      BD=BD+QI*X(K,J)
      BD=QI*BD+X(K,J)


      X(K,JJ)=BD
      CONTINUE
      IF(IBIAS.EQ.0)GO TO 62
      IPWR=IBIAS-1
      DO 61 KK=1,NPT
      TIME=DT*KK
      K=NPT+1-KK
      X(K,NP2)=TIME**IPWR
      CONTINUE
      GO TO 70
      GENERATE UNIT DELAYS
      CONTINUE
      DO 93 I=2,NP1
      I1=I-1
      X(1,I)=0.0
      DO 93 K=2,NPT
      K1=K-1
      X(K,I)=X(K1,I1)
      GO TO 81
      CONTINUE
      SC=1.0
      DO 80 I=2,NP1
      SC=SC*CT
      DO 80 K=1,NPT
      X(K,I)=SC*X(K,I)
```

```
1     CONTINUE
      IF(IPR.LT.4)GO TO 99
      IF(IROUND.EQ.1) WRITE(ILT,178) INT
      IF(IROUND.EQ.0) WRITE(ILT,179) INT
      DO 180 I=1,NP2
80    WRITE(ILT,110)(X(K,I),K=1,NPT)
      WRITE(ILT,1)
9     CONTINUE

110    FORMAT(4(1X,F12.6))
10    FORMAT(10(1X,F7.3))
79    FORMAT(10X,*FILTER,INT=*,I2,*    FOF PROCESSED X*)
78    FORMAT(10X,*FILTER,INT=*,I2,*    NOISY FOF PROCESSED X*)
      FORMAT(/)
      RETURN
      END
```

SUBROUTINE: FIX

PURPOSE: To enhance the Gram matrix of the information signals and to effect noise corrections (Under further development)

EQUATIONS:

$$F = G - \sigma_{est} P$$

where P is the unit noise covariance matrix

ROUTINE VARIABLES

G	Noisy Gram matrix of reverse-time first-order filtered signals
P	Covariance matrix of unit noise (also reverse-time first-order filtered)
C	Corrected matrix
D	Work matrix
N	Dimensionality of the matrices
NC	Not used
SIG	Estimated noise variance
NDIM	Maximum dimensionality of the matrices
IFIX	Option parameter (Use IFIX=1)

FURTHER DESCRIPTION:

Under further development

```
SUBROUTINE FIX(G,P,C,D,X,N,NC,SIG,NDIM,IFIX)
----------------------------------------

ESTIMATE NOISE INTENSITY SIG (ASSUME WHITE NOISE)
CORRECT NOISY MATRIX= C
[P] DENOTES NOISE MATRIX FOR UNIT NOISE
NC IS THE NONZERO SUBMATRIX OF P =COV OF NOISE

DIMENSION G(NDIM,1),P(NDIM,1),C(NDIM,1),D(NDIM,1),X(1)
COMMON /DAO/ISPN,DELTA,SIG2,DT,QI,BIAS,IBIAS,DFAC
COMMON /IO/IR,ILT,IPR,ITFR,IZPR,IROUND,IPLT
DOUBLE PRECISION SUMDET,DT
SI=SIG
IF(IFIX.EQ.0)GO TO 51
GDP=G(1,1)/P(1,1)
JCT=0
SIG=0.0
JCT=JCT+1
SUMDET=0.0
CALL GKROCT(G,D,GDET,X,0,N,NDIM,0)
IF(JCT.EQ.1)DETG=GDET
DO 7 I=1,N
DO 7 J=1,N


SUMDET=SUMDET+D(I,J)*P(I,J)
IF(SUMDET.LT.0.0.AND.IFIX.NE.2)GO TO 11
SI=1.0D0/SUMDET
WRITE(ILT,32)JCT,ICT,GDET,SUMDET,SI
IF(SI/GDP.GT.0.1)WRITE(ILT,31)
ICT=0
CONTINUE
DO 9 I=1,N
DO 9 J=1,N
C(I,J)=G(I,J)-SI*P(I,J)
IF(IFIX.EQ.0)GO TO 11
CALL GKROCT(C,D,CDET,X,0,N,NDIM,0)
IF(CDET.LT.0.0.OR.CDET.GT.GDET)ICT=ICT+1
IF(ICT.GT.5)GO TO 10
IF(ICT.GT.0)SI=SI/2.0
IF(ICT.GT.0)GO TO 51
IF(JCT.GE.5)GO TO 11
THR=DETG*DFAC
IF(JCT.EQ.1)WRITE(ILT,33)DETG,DFAC,THR
SIG=SIG+SI
IF(CDET.GT.THR)CALL MEQUAT(N,N,G,C,NDIM,1)
WRITE(ILT,34)JCT,ICT,GDET,SI,CDET
IF(CDET.GT.THR)GO TO 3
FORMAT(2X,*NOISE VAR EXCESSIVE, SIG/GDP.GT.0.1*)
FORMAT(1X,*J,I GDET,SUMDET,SI:*,2I2,4E11.2)
FORMAT(1X,*GDET,DFAC,THR:*,6E11.2)
FORMAT(1X,*J,I  SI,CDET*,2I2,4E11.2)
RETURN
END
```

SUBROUTINE: GKRDCT

PURPOSE: Basically, this routine finds the cofactors and/or the inverse of a square matrix. It also calculates the denominator parameters through pencil-of-functions method (if ISPN.GE.-1) or the LPC/Prony methods (ISPN.LE.-2)

EQUATIONS:

$$A(z) = D_1^{-1/2} \, (qz)^{-n} \sum_{i=1}^{N} \sqrt{D_i} \, (qz-1)^{N-i}$$

ROUTINE VARIABLES

- X — Gram matrix of information signals
- Y — The adjoint or the inverse matrix of X is returned in Y
- DET — The determinant of X is returned in this variable
- XLAMDA — Vector of computed denominator parameters
- NN — Not used (blank)
- N — Dimensionality of X and Y
- MAX — Maximum dimensionality of X and Y
- IOPT — Option parameter; 0 for finding the inverse and determinant of the matrix X, 1 to find the denominator parameters

FURTHER DESCRIPTION:

Scaling has been introduced in this routine to enable accurate computations even for high order modeling (say 6 to 10).

This routine calls BUILDR to go from D_i to the parameters a_i of A(z).

```
SUBROUTINE GKRDCT(X,Y,DET,XLAMDA,NN,N,MAX,IOPT)
------------------------------------------------------------
DIMENSION XLAMDA(1)
DIMENSION X(MAX,1),Y(MAX,1)
DOUBLE PRECISION A,B,C,D,E
DIMENSION NUM(2,10),SCAL(10),RSC(10),Z(10,10)
DOUBLE PRECISION DT,SC,AD,BD
COMMON /DAO/ISPN,DELTA,SIG2,DT,QI,BIAS,IBIAS,DFAC
COMMON /IO/IR,ILT,IPR,ITPR,IZPR,IROUND,IPLT
IGKR=1
IF(N.NE.1)GO TO 3
Y(1,1)=1.0/X(1,1)
DET=X(1,1)
GO TO 61
CONTINUE
NMAT=N
LPC=0
IF(ISPN.LE.-2.AND.IOPT.EQ.1)LPC=1
IF(LPC.EQ.1)NMAT=N-1
------SCALE---
DO 11 I=1,NMAT
SCAL(I)=1.0
IF(X(I+LPC,I+LPC).GT.0.1E-20)SCAL(I)=SQRT(X(I+LPC,I+LPC))
RSC(I)=1.0/SCAL(I)
CONTINUE
DO 6 I=1,NMAT
DO 6 J=1,NMAT
Y(J,I)=X(J+LPC,I+LPC)*RSC(I)*RSC(J)
Z(J,I)=Y(J,I)
IF(ITPR.GE.1)CALL PRTMAT(Z,NMAT,NMAT,10,0)

---BEGIN GK REDUCTION----
A=1.0
DO 43 I=1,NMAT
B=0.0
L=I
M=I

     FIND LARGEST ENTRY A(L,M) IN THE LOWER DIAGONAL SUBMATRIX

DO 18 J=I,NMAT
DO 18 K=I,NMAT
IF(ABS(Y(K,J)).LE.DABS(B))GO TO 18
B=ABS(Y(K,J))
L=K
M=J
CONTINUE

    INTERCHANGE ROWS

IF(L.EQ.I)GO TO 24
DO 23 J=1,NMAT
C=Y(L,J)
Y(L,J)=Y(I,J)
Y(I,J)=C

   INTERCHANGE COLUMNS

IF(M.EQ.I)GO TO 29
```

```
      DO 28 J=1,NMAT
      C=Y(J,M)
      Y(J,M)=Y(J,I)
8     Y(J,I)=C
9     NUM(1,I)=L
      NUM(2,I)=M
      B=Y(I,I)
      Y(I,I)=A
      DO 42 J=1,NMAT
      IF(J.EQ.I)GO TO 42
      C=-Y(I,J)
      Y(I,J)=0.0
      DO 41 K=1,NMAT
      D=C*Y(K,I)
      E=B*Y(K,J)+D
      IF(DABS(E).LT.1.0D-10*DABS(D))E=0.0
1     Y(K,J)=E/A
2     CONTINUE
3     A=B

        RESTORE COLUMNS

      DO 58 I=2,NMAT
      J=NMAT+1-I
      K=NUM(2,J)
      IF(K.EQ.J)GO TO 52
      DO 51 L=1,NMAT
      C=Y(K,L)
      Y(K,L)=Y(J,L)
      Y(J,L)=C
      K=NUM(1,J)

         RESTORE ROWS

      IF(K.EQ.J)GO TO 58
      DO 57 L=1,NMAT
      C=Y(L,K)
      Y(L,K)=Y(L,J)
      Y(L,J)=C
      CONTINUE
      DET=A
      DSCAL=1.0
      DO 59 I=1,NMAT
      DSCAL=SCAL(I)*DSCAL
      DET=DET*SCAL(I)*SCAL(I)
      IF(ITPR.GE.1) WRITE(ILT,337)DET,A,(RSC(I),I=1,NMAT)
7     FORMAT(1X,*DET,A, RSC(I)= *,7E11.4)
      IF(IOPT.EQ.1.AND.ISPN.GE.-1)GO TO 61
      IF(ITPR.GE.1) CALL PRTMAT(Y,N,N,MAX,0)
      IF(ITPR.GE.1) CALL PROMAT(Z,Y,NMAT,10,MAX,0)
      DO 60 I=1,NMAT
      DO 60 J=1,NMAT
      Y(I,J)=Y(I,J)*RSC(I)*RSC(J)/A
      CONTINUE
      IF(IOPT.NE.1)GO TO 1000
      IF(ISPN.LE.-2)GO TO 440
      IF(Y(1,1).LT.0.0)GO TO 1000
      .....
      SC=1.0
      DO 200 I=2,NMAT
      SC=SC*CT
      PSC(I)=RSC(I)/RSC(1)
      IF(IGKR.EQ.0) XLAMDA(I)=RSC(I)*Y(I,1)/Y(1,1)
      IF(IGKR.EQ.0)GO TO 199
```

```
      A=Y(I,I)
      IF(Y(I,I).LT.0.0)A=0.0
      IF(IGKR.EQ.2)A=ABS(Y(I,I))
      XLAMDA(I)=RSC(I)*DSQRT(A/Y(1,1))
      IF(Y(I,1).LT.0.0)XLAMDA(I)=-XLAMDA(I)
 99   XLAMDA(I)=SC*XLAMCA(I)
 00   CONTINUE
      XLAMDA(1)=1.0D0
      AT=RSC(1)*DSCAL*SQRT(Y(1,1))
      IF(IPR.GE.1)WRITE(6,106)(XLAMDA(I),I=1,NMAT),AT
      NPP=N
      IF(IBIAS.NE.0)NPP=N-1
      CALL BUILDR(Y,XLAMDA,NPP,MAX)
 16   FORMAT(5X,*SYNTHETIC VECTOR, AND SQRT(Y11)*,/,10G12.5)
      GO TO 1000
 .0   CONTINUE
      IF(ITPR.LE.0)GO TO 449
      WRITE(ILT,448)
      CALL PRTMAT(Y,N,N,MAX,0)
      DO 442 I=1,NMAT
      DO 442 J=1,NMAT
 .2   Z(I,J)=X(I+1,J+1)
      CALL PROMAT(Z,Y,NMAT,10,MAX,0)
  7   FORMAT(2X,9G11.4)
  8   FORMAT(2X,* INVERSE AND PRODUCT MATRICES*)
  9   CONTINUE
      XLAMDA(1)=1.0
      DO 450 I=2,N
      XLAMDA(I)=0.0
      DO 450 J=1,NMAT
  0   XLAMDA(I)=XLAMDA(I)-Y(I-1,J)*X(J+1,1)
  00  CONTINUE
      RETURN
      END
```

SUBROUTINE MEQUAT

PURPOSE: To set an m x n dimensional matrix B equal to a matrix A of the same dimensionality

Equation: B = A

ROUTINE VARIABLES:

	M	Row dimensionality of B and A
	N	Column dimensionaltiy of B and A
	B	Matrix to be set
	A	Matrix to which the matrix B is set
	NDIM	Maximum number of rows permissible
	IOPT	Print option; 0 for no printing, 2 or greater for printing

```
SUBROUTINE MEQUAT(M,N,B,A,NDIM,IOPT)
-----------------------------------
IOPT=0   SET B TO ZERO
     1      B EQUAL TO A
     10     B TO IDENTITY
DIMENSION A(NDIM,1),B(NDIM,1)
DO 33 I=1,M
DO 33 J=1,N
IF(IOPT.NE.1) B(I,J)=0.0


IF(IOPT.EQ.10.AND.I.EQ.J)B(I,J)=1.0
IF(IOPT.EQ.1) B(I,J)=A(I,J)
CONTINUE
RETURN
END
```

SUBROUTINE: PLOP

PURPOSE: To plot a pair of columns of the array X (This routine may be substituted by user's own routine)

```
SUBROUTINE PLOP(NPT,NF,Y,NDIM,T0,DT,LABEL,INDEP,IBUF)
-----------------------------------------------
NPT=NUMB OF TIME PTS  (WARNING: NDIM SHOULD BE.GE.NPT+2)
NF=NUMBER OF FUNS
Y(K, ) DATA ARRAY OF DIMENSION NDIM,NF
T0=INITIAL TIME, DT=TIME INCREMENT
LABEL, INDEP = TITLES FOR Y AND X AXES
DIMENSION Y(NDIM,NF),YY(2),LABEL(1),INDEP(1)
DIMENSION X(512),IBUF(512)
COMMON /IO/IR,ILT,IPR,ITPR,IZPR,IROUND,IPLT
M=NF*NDIM
M1=M+1
M2=M+2
NPT1=NPT+1
NPT2=NPT+2
X(1)=T0
DO 9 K=2,NPT
X(K)=X(K-1)+DT
DO 8 I=1,NF
DO 8 K=NPT1,NDIM
Y(K,I)=Y(NPT,I)

INITIALIZE(LIQ.INK, 12IN.PAPER)     MAX.LENGTH=60IN
CALL PLOTMX(60.0)
SET ORIGIN
CALL PLOT(0.,-.5,3)
CALL FACTOR(5.0/6.5)
BEGIN PLOTTING
CALL SCALE(X,6.5,NPT,1)
CALL SCALE(Y(1,1),10.0,M,1)
CALL AXIS(0.,0.,11HTIME (SEC.),
*-16,6.5,0.,X(NPT1),X(NPT2))
CALL AXIS(0.,0.,16HRESPONSES Y    ,Y,
*16,10.,90.,Y(M1),Y(M2))
WRITE(6,6)X(NPT1),X(NPT2)
WRITE(6,7)Y(M1),Y(M2)
FORMAT(1X,*T0,DIV (6.5 DIV)*,4(1X,F7.3))
FORMAT(1X,*Y0,DIV (10 DIV)*,4(1X,F7.3))
DO 10 I=1,NF
Y(NPT1,I)=Y(M1)
Y(NPT2,I)=Y(M2)
IF(I.EQ.1)CALL LINE(X,Y(1,I),NPT,1,I-1,I)
IF(I.EQ.2)CALL DASHLN(X,Y(1,2),NPT,1)
CONTINUE
CALL PLOT(10.,0.,-3)
RETURN
END
```

SUBROUTINE: POLCON

PURPOSE: To combine the factors of a polynomial in order to produce the coefficients.

```
SUBROUTINE POLCON(C,R2,K,N)
-------------------------
A POLYINOMIAL CONSTRUCTION PROGRAM NEEDED FOR ZTOS
DIMENSION C(1),R2(1)
COMPLEX C,R2,COMP
DIMENSION DC(2)
EQUIVALENCE (COMP,DC)
NP1=N+1
DO10I=2,NP1
R2(I)=0.0000
R2(1)=1.0000
DO4I=1,N


COMP=C(I)
IF(I.EQ.K.OR.(DC(1).EQ.0.000.AND.DC(2).EQ.0.000))GO TO 4
DO2JJ=1,I
J=I-JJ+1
R2(J+1)=R2(J+1)*C(I)+R2(J)
R2(1)=R2(1)*C(I)
CONTINUE
RETURN
END
```

EQUATION:

$$(x - c_1)(x - c_2) \;....\; (x - c_n)$$

$$= r_1 + r_2x + \;...\; + r_nx^{n-1} + r_{n+1}x^n$$

VARIABLES:

- C Vector containing the roots of the factors
- R2 Vector returning the coefficients of the polynomial
- K Exclude Kth factor if $K \neq 0$
- N Number of roots contained in C

SUBROUTINE: POLRT

PURPOSE: To find the roots of a polynomial

EQUATIONS:

$$a_1 + a_2x + \ldots. + a_{n+1}x^n$$

$$(x - p_1 - jq_1)(x - p_2 - jq_2) \ldots. (x - p_n - jq_n)$$

ROUTINE VARIABLES

XCOF	Coefficients of the polynomial (XCOF(1)= a_1)
COF	Work vector
M	Order of polynomial
ROOTR	Real parts of the roots are returned in this vector
ROOTI	Imaginary parts of the roots are returned in this vector
IER	Type of error, if any, returned in this integer variable

FURTHER DESCRIPTION:

```
      SUBROUTINE POLRT(XCOF,COF,M,ROOTR,ROOTI,IER)
     ------------------------------------------------------------
 
              COMPUTES THE REAL AND COMPLEX ROOTS OF A REAL POLYNOMIAL
 
          DESCRIPTION OF PARAMETERS
              XCOF -VECTOR OF M+1 COEFFICIENTS OF THE POLYNOMIAL
                    ORDERED FROM SMALLEST TO LARGEST POWER
              COF  -WORKING VECTOR OF LENGTH M+1
              M    -ORDER OF POLYNOMIAL
              ROOTR-RESULTANT VECTOR OF LENGTH M CONTAINING REAL ROOTS
                    OF THE POLYNOMIAL
              ROOTI-RESULTANT VECTOR OF LENGTH M CONTAINING THE
                    CORRESPONDING IMAGINARY ROOTS OF THE POLYNOMIAL
              IER  -ERROR CODE WHERE
                    IER=0  NO ERROR
                    IER=1  M LESS THAN ONE
                    IER=2  M GREATER THAN 36
                    IER=3  UNABLE TO DETERMINE ROOT WITH 500 INTERATIONS
                           ON 5 STARTING VALUES
                    IER=4  HIGH ORDER COEFFICIENT IS ZERO
 
      DIMENSION XCOF(1),COF(1),ROOTR(1),ROOTI(1)
 
 
      DOUBLE PRECISION XO,YO,X,Y,XPR,YPR,UX,UY,V,YT,XT,U,XT2,YT2,SUMSQ,
     1 DX,DY,TEMP,ALPHA,XCOF,COF,ROOTR,ROOTI,ER1,ER2,XSS,XS,YSS,YS,TOL
      COMMON /IO/IR,ILT,IPR,ITPR,IZPR,IROUND,IPLT
 
              LIMITED TO 36TH ORDER POLYNOMIAL OR LESS.
              FLOATING POINT OVERFLOW MAY OCCUR FOR HIGH ORDER
              POLYNOMIALS BUT WILL NOT AFFECT THE ACCURACY OF THE RESULTS.
 
          METHOD
              NEWTON-RAPHSON ITERATIVE TECHNIQUE.  THE FINAL ITERATIONS
              ON EACH ROOT ARE PERFORMED USING THE ORIGINAL POLYNOMIAL
              RATHER THAN THE REDUCED POLYNOMIAL TO AVOID ACCUMULATED
              ERRORS IN THE REDUCED POLYNOMIAL.
 
      ER2=1.0D+50
      TOL=1.0D-8
      IFIT=0
      N=M
      IER=0
      IF(XCOF(N+1)) 10,25,10
   10 IF(N) 15,15,32
 
          SET ERROR CODE TO 1
 
   15 IER=1
   20 IF(IER)200,201,200
    0 WRITE(6,203)IER
    3 FORMAT(1X,*ERROR CALLED FROM POLRT, IER = *,I3)
    . RETURN
 
          SET ERROR CODE TO 4
 
   25 IER=4
      GO TO 20
 
          SET ERROR CODE TO 2
```

```
 30 IER=2
    GO TO 20
 32 IF(N-36) 35,35,30
 35 NX=N
    NXX=N+1
    N2=1
    KJ1 = N+1
    DO 40 L=1,KJ1
    MT=KJ1-L+1
 40 COF(MT)=XCOF(L)

        SET INITIAL VALUES

 45 XO=.00500101
    YO=0.01000101

        ZERO INITIAL VALUE COUNTER

    -------BEGIN ITERATION-------------------
    X AND Y ARE THE REAL AND IMAG PARTS OF ROOT
    IN=0
 50 X=XO

        INCREMENT INITIAL VALUES AND COUNTER

    XO=-10.0*YO
    YO=-10.0*X



        SET X AND Y TO CURRENT VALUE

    X=XO
    Y=YO
    IN=IN+1
    GO TO 59
 55 IFIT=1
    XPR=X
    YPR=Y

        EVALUATE POLYNOMIAL AND DERIVATIVES

 59 ICT=0
 60 UX=0.0
    UY=0.0
    V =0.0
    YT=0.0
    XT=1.0
    U=COF(N+1)
    IF(U) 65,130,65
 65 DO 70 I=1,N
    L =N-I+1
    TEMP=COF(L)
    XT2=X*XT-Y*YT
    YT2=X*YT+Y*XT
    U=U+TEMP*XT2
    V=V+TEMP*YT2
    FI=I
    UX=UX+FI*XT*TEMP
    UY=UY-FI*YT*TEMP
    XT=XT2
```

```
 70 YT=YT2
    SUMSQ=UX*UX+UY*UY
    IF(SUMSQ) 75,110,75
 75 DX=(V*UY-U*UX)/SUMSQ
    X=X+DX
    DY=-(U*UY+V*UX)/SUMSQ
    ILOC=75
    IF(ITPR.GE.1) WRITE(ILT,442) ILOC
    Y=Y+DY
    XSS=X
    YSS=Y
    IF(YSS.EQ.0.0D0)YSS=1.0D0
    IF(XSS.EQ.0.0D0)XSS=1.0D0
    RMAG=SQRT(XSS*XSS+YSS*YSS)
    MODIF.APR.80   *SS TO *SS+.00001*RMAG IN NEXT CARD
    ER1=ABS(DX/(XSS+.00001*RMAG))+ABS(DY/(YSS+.00001*RMAG))
    IF(ITPR.GE.1) WRITE(ILT,444)DX,XSS,DY,YSS,ER1,ER2
    ILOC=77
    IF(ITPR.GE.1) WRITE(ILT,442) ILOC
    IF(ER1.GT.ER2)GO TO 78
    ER2=ER1
    XS=XSS
    YS=YSS
    IF(ER1-TOL)100,80,80

       STEP ITERATION COUNTER

 80 ICT=ICT+1
    IF(ICT-500) 60,85,85
 85 IF(IFIT)100,90,100
 90 IF(IN-5) 50,95,95
    ---------------EXIT FROM ITERATIONS-----------



       SET ERROR CODE TO 3

 95 IER=3
    X=XS
    Y=YS
    ER1=ER2
100 DO 105 L=1,NXX
    MT=KJ1-L+1
    TEMP=XCOF(MT)
    XCOF(MT)=COF(L)
105 COF(L)=TEMP
    ITEMP=N
    N=NX
    NX=ITEMP
    IF(IFIT) 120,55,120
110 IF(IFIT) 115,50,115
115 X=XPR
    Y=YPR
120 IFIT=0
?   IF(ABS(Y)-1.0D-8*ABS(X))135,125,125
125 ALPHA=X+X
    SUMSQ=X*X+Y*Y
    N=N-2
    GO TO 140
130 X=0.0
    NX=NX-1
    NXX=NXX-1
```

```
 135 Y=0.0
     SUMSQ=0.0
     ALPHA=X
     N=N-1
 140 COF(2)=COF(2)+ALPHA*COF(1)
 145 DO 150 L=2,N
 150 COF(L+1)=COF(L+1)+ALPHA*COF(L)-SUMSQ*COF(L-1)
 155 ROOTI(N2)=Y
     ROOTR(N2)=X
     IF(ER1.GT.TOL)WRITE(6,554)N2,ER1
 554 FORMAT(1X,*ERROR ON *,I3,* TH ROOT IS *,D10.3)
     ER2=1.0D+50
     N2=N2+1
     IF(SUMSQ) 160,165,160
 160 Y=-Y
     SUMSQ=0.0
     GO TO 155
 165 IF(N) 20,20,45
 2   FORMAT(2X,*TEST IN POLRT*,I5)
 4   FORMAT(1X,*DX,XSS,DY,YSS,ER1-2*,2E8.1,2X,2E8.1,2X,2E9.2)
     RETURN
     END
```

SUBROUTINE: PRDMAT

PURPOSE: This subroutine computes the product of two square matrices.

EQUATIONS: $A \leftarrow A*B$

ROUTINE VARIABLES:

ROUTINE VARIABLES:	A, B	N x N matrices
	NDIM1	Maximum row dimension of A
	NDIM2	Maximum row dimension of B
	LOC	An integer which is printed

```
SUBROUTINE PROMAT(A,B,N,NDIM1,NDIM2,LOC)
------------------------

COMPUTES PRODUCT A = A*B

DIMENSION A(NDIM1,1),B(NDIM2,1),C(10,10)
IF(LOC.GE.1)WRITE(6,5)LOC
DO 31 I=1,N
DO 31 J=1,N
C(I,J)=0.0
DO 21 K=1,N
C(I,J)=C(I,J)+A(I,K)*B(K,J)
CONTINUE
DO 41 I=1,N
DO 41 J=1,N
A(I,J)=C(I,J)
DO 45 I=1,N
WRITE(6,15)(A(I,J),J=1,N)
FORMAT(*    LOCATION/INTEGER=*,I5)
FORMAT(2X,10G13.6)
RETURN
END
FUNCTION COMB(N,M)
CALCULATES COMBINATION M OUT OF N
IF(N.LE.0)GO TO 99
L=1
LD=1
IF(M.EQ.0)GO TO 8
MN1=N-M+1
DO 5 I=MN1,N


L=L*I
DO 7 I=1,M
LD=LD*I
COMB=L/LD
RETURN
END
```

SUBROUTINE: PRTMAT, PRTVEC, PRCVEC, PRVEC

PURPOSE: These four subroutines perform printing of arrays, PRTMAT of matrices and the other three of vectors. See subroutines for comments.

EQUATIONS:

ROUTINE VARIABLES

Subroutine PRTMAT

A	Matrix being printed
M	Its row dimensionality
N	Its column dimensionality
NDIM	Maximum number of rows permitted
LOC	Use LOC=0. If nonzero, this same number is printed.

FURTHER DESCRIPTION:

```
      SUBROUTINE PRTMAT(A,M,N,NDIM,LOC)
     ------------------------
      PRINTS A MATRIX, AND AN INTEGER (PERHAPS A LOCATION) IF LOC.GE.1

      DIMENSION A(NDIM,1)
      IF(LOC.GE.1)WRITE(6,5)LOC
      DO 31 I=1,M
      WRITE(6,15)(A(I,J),J=1,N)
      FORMAT(*    LOCATION/INTEGER=*,I5)

      FORMAT(2X,10G13.6)
      RETURN
      END

      SUBROUTINE PRTVEC(A,N)
     --------------------------
         PRINTS A COMPLEX VECTOR, MAX N=5
         WHEN VARIABLE IS SINGLE PRECISION

      COMPLEX A(5)
      WRITE(6,1)(A(I),I=1,N)
      FORMAT(1X,2E12.5,4(3X,2E12.5))
      RETURN
      END
      SUBROUTINE PRCVEC(A,N)
     --------------------------
         PRINTS A COMPLEX VECTOR, MAX N=5
         WHEN VARIABLE IS DOUBLE PRECISION

      COMPLEX A
      DIMENSION A(1)
      WRITE(6,1)(A(I),I=1,N)
      FORMAT(1X,2D12.5,4(3X,2D12.5))
      RETURN
      END
      SUBROUTINE PRVEC(A,N)
     --------------------------
      THIS SUBROUTINE OUTPUTS DOUBLE PRECISION SINGLE DIMENSIONED ARRAY
      DIMENSION A(1)
      WRITE(6,1)(A(I),I=1,N)
      FORMAT(1X,6D16.6)
      RETURN
      END
```

SUBROUTINE: RESPON

PURPOSE: To determine the response of the digital transfer function H(z) to an input sequence v(k). The coefficients of H(z) are given to the routine in the NPNP2=N+N+2 vector GAMMA

EQUATIONS:

$$H(z) = \frac{b_0 + b_1 z^{-1} + \dots + b_n z^{-n}}{1 + a_1 z^{-1} + \dots + a_n z^{-n}}$$

$$x_k = -a_1 x_{k-1} - \dots - a_n x_{k-n} + b_0 v_k + \dots + b_n v_{k-n}$$

ROUTINE VARIABLES

X	The vector which returns the response of H(z)
V	Vector containing the input sequence
N	order of transfer function H
GAMMA	Vector of coefficients of H GAMMA = $(1, a_1, \dots, a_n, -b_0, -b_1, \dots, -b_n)$
XLAMDA	Work vector
MP1	Number of response points generated

FURTHER DESCRIPTION:

The routine assumes zero initial conditions

```
      SUBROUTINE RESPON(X,V,N,GAMMA,XLAMDA,MP1)
-----------------------------------------------------------------
      DIMENSION X(1),V(1),GAMMA(1),XLAMDA(1)
      DOUBLE PRECISION XSAV,AC,BD
      NM1=N-1
      NP1=N+1
      NPNP1=N+N+1
      NPNP2=N+N+2
      DO 19 I=1,NPNP1
      XLAMDA(I)=0.0
      XSAV=0.0
      DO 20 K=1,MP1
      IF(N.EQ.1)GO TO 25
      DO 21 I=1,NM1
      J=NP1-I
      XLAMDA(J)=XLAMDA(J-1)
      CONTINUE
      DO 22 I=1,N
      J=NPNP2-I
      XLAMDA(J)=XLAMDA(J-1)
      XLAMDA(1)=XSAV
      XLAMDA(NP1)=V(K)
      XSAV=0.0
      DO 23 I=1,NPNP1
      XSAV=XSAV-GAMMA(I+1)*XLAMDA(I)
      IF(DABS(XSAV).GE.1.0E10)XSAV=0.0
      X(K)=XSAV
      RETURN
      END
```

SUBROUTINE: SIGNAL

PURPOSE: This routine generates a signal which is a weighted sum of exponential*sinusoid terms

EQUATIONS:

$$f(t) = \sum_{i=1}^{m} W_i \, e^{-\alpha_i t} \, \mathrm{Sin}(\beta_i t + \phi_i)$$

$$F(k) = f(k\Delta)$$

ROUTINE VARIABLES

F	Vector returning the generated signal
NPT	Number of signal points generated
AMP	Vector of weights associated with each exp*sinusoid term
SR	Vector of exponents "
SI	Vector of radian frequencies "
SPH	Vector of phases "
DT	Sampling interval
NCOMP	Number of terms

FURTHER DESCRIPTION:

This routine is useful only in the simulation mode and is called when ISIM=2

```
SUBROUTINE SIGNAL(F,NPT,AMP,SR,SI,SPH,DT,NCOMP)
------------------------------------------------
DIMENSION F(1),AMP(1),SR(1),SI(1),SPH(1)
COMMON /IO/IR,ILT,IPR,ITPR,IZPR,IROUND,IPLT
DOUBLE PRECISION A,B,C,X
DO 12 K=1,NPT
F(K)=0.0
DO 20 I=1,NCOMP
A=SR(I)*DT
B=SI(I)*DT
C=SPH(I)
DO 15 KK=1,NPT
K=KK-1
X=AMP(I)
IF(A.NE.0.0)X=X*DEXP(A*K)
IF(B.NE.0.0)X=X*DSIN(B*K+C)
F(KK)=X+F(KK)
CONTINUE

IF(IPR.LT.2)GO TO 30
WRITE(ILT,9)
WRITE(ILT,6)(F(K),K=1,NPT)
WRITE(ILT,1)
CONTINUE
FORMAT(/)
FORMAT(20(1X,F5.2))
FORMAT(10X,* F SIGNAL*)
RETURN
END
```

SUBROUTINE: VEQUAT

PURPOSE: To equate a vector Y to another suitable vector

EQUATIONS:

Y(k) =	0 vector		if IOPT=0
=	X(k)		=1 or 2 (print also)
=	Y(k)*X(1)		=3
=	Y(k)+X(1)		=4
=	δ_k	(1,0,0,0,..)	=10
=	1	(1,1,1,1,..)	=11

VARIABLES:

NPT	Dimensionality of X
Y	The vector to be set
X	Auxiliary vector
NPUL	Not used
IOPT	Option parameter (see above)

```
 SUBROUTINE VEQUAT(NPT,Y,X,NPUL,IOPT)
-----------------------------------
 IOPT=0   SET   Y  TO ZERO
      1 OR 2  SET  Y=X   (PRINT IF 2)
      3  SET Y= Y* CONST X(1)
      4  SET Y= Y+ CONST X(1)
      9  SET  Y  TO ZERO
      10 SET  Y=IMPULSE
      11 SET  Y=STEP
 DIMENSION X(1),Y(1)
 IF(IOPT.EQ.0) IOPT=9
 DO 33 K=1,NPT
 IF(IOPT.EQ.1.OR.IOPT.EQ.2)Y(K)=X(K)
 IF(IOPT.EQ.3)Y(K)=Y(K)*X(1)
 IF(IOPT.EQ.4)Y(K)=Y(K)+X(1)
 IF(IOPT.GE.9)Y(K)=0.0
 IF(IOPT.EQ.11)Y(K)=1.0
 CONTINUE
 IF(IOPT.EQ.2) WRITE(6,6)(Y(K),K=1,NPT)
 FORMAT(2X,10G12.5)
 IF(IOPT.EQ.10)Y(1)=1.0
 RETURN
 END
```

SUBROUTINE: ZTOS

PURPOSE: Given the z-domain transfer function H(z) this routine computes the equivalent s-domain transfer function H(s)

EQUATIONS:

H(z)	H(s)	Impulse invariant if	IZTS=0
		Pulse invariant "	=1
		Trapezoid invariant "	=2

ROUTINE VARIABLES

B Vector of denominator parameters

A Vector of numerator parameters

Comment - unfortunately, these names are opposite to the convention used in the rest of the report. However, the reader need not be concerned about this unless he wishes to study this routine; in the latter case, he should bear this in mind.

N Order of the transfer function

DELTA Sampling interval

IZTS Option for type of conversion (see above)

FURTHER DESCRIPTION:

See Gold and Rader, or Oppenheim and Schafer, or Stanley for the theory of z-domain to s-domain converison.

```
      SUBROUTINE ZTOS(B,A,N,DELTA,IZTS)
     ----------------------------------------------------------------

        GIVEN THE DISCRETE DESCRIPTION THIS SUBROUTINE COMPUTES THE
        EQUIVALENT CONTINUOUS DOMAIN DESCRIPTION OF A LINEAR DYNAMIC
        SYSTEM
        THE INPUT ARRAYS A AND B ARE FILLED ACCORDING TO THE DIFFERENCE
        EQUATION
             B(1)*Y(K)+B(2)*Y(K-1)+...+B(N+1)*Y(K-N)
                  -A(1)*U(K)-A(2)*U(K-1)-...-A(N+1)*U(K-N) = 0
        B(1) MUST EQUAL 1

        POLES OF THE CONTINUOUS DOMAIN MUST BE DISTINCT AND NON-ZERO
        FOR THE TRNSFORMATION TO BE VALID



        UPON RETURNING ARRAYS A AND B CONTAIN THE EQUIVALENT COTINUOUS
        DESCRIPTION STORED ACCORDING TO THE DIFFERENTIAL EQUATION
             B(1)*Y(T)+B(2)*D(1,Y(T))+...+B(N+1)*D(N,Y(T))
                  -A(1)*U(T)-A(2)*D(1,U(T))-...-A(N+1)*D(N,U(T)) = 0
        WHERE D(M,F(T)) = THE MTH TIME DERIVATIVE OF FUNCTION, F

        B(N+1) ALWAYS IS 1

        N = ORDER OF SYSTEM
        N (MAXIMUM) = ONE LESS THAN THE DIMENSION SUBSCRIPT


        IZTS=0 ---> IMPULSE
        IZTS=1 ---> PULSE
        IZTS=2 ---> TRAPAZOIDAL
        DELTA = SAMPLING INTERVAL = 1/(SAMPLING FREQUENCY)

      COMPLEX CR,CA,CF,CG,CF1,CON1,CON2,CONT
      COMPLEX POLE(20),ZRO(20)
      DIMENSION B(1),A(1),TEMP(20),RR(20),RI(20),CR(20),CA(20),CF(20),
     1CG(20),CF1(20),ZR(20),ZI(20)
      COMMON /IO/IR,ILT,IPR,ITPR,IZPR,IROUND,IPLT
      CONT=0.0000
      NP1=N+1
      IF(A(NP1))410,411,410
    0 ICHECK=2
      GO TO 401
    1 ICHECK=1
      GO TO 400
    1 CONT=A(NP1)/B(NP1)
      DO402I=1,N
    : A(I)=A(I)-CONT*B(I)
      IF(ITPR.GE.1) WRITE(ILT,441)
    ; CALL POLRT(B,TEMP,N,RR,RI,IER)
      IF(ITPR.GE.1) WRITE(ILT,442)IER
    . FORMAT(2X,*GOING INTO POLRT*)
    : FORMAT(2X,*RETURNED FROM POLRT,  IER=*,I2)
      DO6I=1,N
      CR(I)=CMPLX(RR(I),RI(I))
      CF(I)=1.0000/CR(I)
      IF(IZPR.GE.1) WRITE(6,1002)
    2 FORMAT(* THE POLES OF THE Z-DOMAIN*)
      IF(IZPR.GE.1)CALL PRCVEC(CF,N)
      IF(IZPR.EQ.1)GO TO 1101
      DO3I=1,N
      CON1=1.0000
```

```
      CON2=0.0D00
      DO4J=1,N
      CON2=CON2*CR(I)+A(N-J+1)
      IF(I-J)5,4,5
      CON1=CON1*(1.0D00-CR(I)*CF(J))
      CONTINUE
      CA(I)=CON2/CON1
      IF(IZTS-1)225,224,230
25    DO222I=1,N
      CA(I)=CA(I)/DELTA
22    CR(I)=CLOG(CR(I))/DELTA
      GO TO 226
24    DO2I=1,N
      CON1=CLOG(CR(I))/DELTA
      CA(I)=CA(I)*CR(I)*CON1/(CR(I)-1.0D00)
      CR(I)=CON1
      GO TO 226


0     ICHECK=2
      DO231I=1,N
      CON1=CLOG(CR(I))/DELTA
      CON2=CA(I)*CR(I)/((1.0D00-CR(I))*(1.0D00-CR(I)))
      CONT=CONT-CON2*(1.0D00+CON1*DELTA-CR(I))
      CA(I)=CON2*CON1*CON1*DELTA
1     CR(I)=CON1
      WRITE(6,2131)
31    FORMAT(2X,*TIME SCALED FOR SCATTERER*)
6     WRITE(6,1004) IZTS
04    FORMAT(1X,*S-POLES*,5X,*SR*,9X,*SI*,9X,*SMAG*,9X,*FR*,7X,
     +*IZTS=*,I2)
      DT=0.001953125
      RDT=DELTA/DT
      RDT=1.0
      DO 228 I=1,N
      SSR=-REAL(CR(I))*RDT
      SSI=AIMAG(CR(I))*RDT
      SSM=CABS(CR(I))*RDT
      SSFR=(SSM/6.2831853)
8     WRITE(6,1010) I,SSR,SSI,SSM,SSFR
      IF(IPR.LE.0)GO TO 1101
10    FORMAT(3X,I2,5F12.4)
      IF(IPR.GE.1)WRITE(6,1003)
03    FORMAT(* NUMERATOR CONSTANTS OF FACTORIZED H(S)*)
      IF(IPR.GE.1)CALL PRCVEC(CA,N)
      DO 240 I=1,N
)     POLE(I)=CR(I)
      CALL POLCON(CR,CG,0,N)
      DO7I=1,NP1
      CF(I)=0.0D00
      DO9K=1,N
      CALL POLCON(CR,CF1,K,N)
      DO9J  ,N
      CF(J)=CF(J)+CF1(J)*CA(K)
      CF(NP1)=0.0D00
      GO TO (403,404),ICHECK
      CF(NP1)=CONT
      DO405I=1,N
      CF(I)=CF(I)+CONT*CG(I)
      CONTINUE
      IF(IPR.LE.1)GO TO 520
```

```
      FIND ZEROS

      DO 507 I=1,NP1
07      A(I)=CF(I)
      NC=0
      DO 510 I=1,NP1
        IF(ABS(A(I)).GT.1.D-6)NC=I-1
10    CONTINUE
      IF(NC.EQ.0)GO TO 520

      N1=NO+1
      AK=A(N1)
      DO 515 I=1,N1
15      A(I)=A(I)/AK
      CALL PCLRT(A,TEMP,NC,RR,RI,IER)
      DO 517 I=1,NO
      ZR(I)=RR(I)
      ZI(I)=RI(I)
      ZRO(I)=CMPLX(ZR(I),ZI(I))
.7    CONTINUE
      IF(IPR.GE.2)WRITE(6,1007)AK

07        FORMAT(* ZEROS OF H(S), NUMERATOR CONSTANT =*,E13.6)
      IF(IPR.GE.2)CALL PRTVEC(ZRO,NO)

0     CONTINUE
      DO20I=1,NP1
      B(I)=CG(I)
      A(I)=CF(I)
      IF(IPR.GE.1)WRITE(6,1005)
05    FORMAT(* S-DOMAIN DENOMINATOR*)
      IF(IPR.GE.1)CALL PRVEC(B,NP1)
      IF(IPR.GE.1)WRITE(6,1006)
06    FORMAT(* S-DOMAIN NUMERATOR*)
      IF(IPR.GE.1)CALL PRVEC(A,NP1)
01    RETURN
      END
```

APPENDIX B

Modeling of a Noisy Test Signal

The signal considered in example 1 of Section IV is

$$x(k) = y(k) + w(k)$$

where $y(k)$ is the impulse response of $(1 - 1.92z^{-1} + z^{-2})/(1 - 2.68z^{-1} + 2.476z^{-2} - 0.782z^{-3})$ and $w(k)$ is additive white noise. The true signal of interest, $y(k)$, is shown in Fig. B1 and the signal under test, $x(k)$, is shown in Fig. B2.

Given below are the (card deck) input to program POF-FILTER and, succeeding it, the printer output from the program.

INPUT CARDS

```
                    EXAMPLE 1    LPC, PRONY   VS   PENCIL-OF-FNS.
NPT       IPLSP IPLT   XMSE                     BIAS               SIG2
     N   NCOMP NNPT              DT                       ANBIAS
0100       5 1+2 1 100 -5.000000 +1.000000 +0.000000 +0.000000 +0.001
+1.000000 -2.680000 +2.476000 -0.782000
+1.000000 -1.920000 +1.000000
IPR IREM ISPN IFIX NFIX    -- PENCIL-OF-FUNCTIONS--
 2-1 1-1-1 1+0 1 100 +0.8
IPR IREM ISPN IFIX NFIX    -- PENCIL-OF-FUNCTIONS:  ENHANCED--
 2-1 1-1 1 1+0 1 100 +0.000000 +0.00011
IPR IREM ISPN IFIX NFIX    -- LPC:  AUTOCORR--
 2-1 1-3-1 1+0 1 100 +1.0
IPR IREM ISPN IFIX NFIX    -- LPC:  COV--
 2-1 1-2-1 1+0 1 100 +1.0
IPR IREM ISPN IFIX NFIX    -- PRONY, AUTOCORR--
 2-1 1-3-1 1+0 1 100 +1.0
IPR IREM ISPN IFIX NFIX    -- PRONY , COV--
 2-1 1-2-1 1+0 1 100 +1.0
```

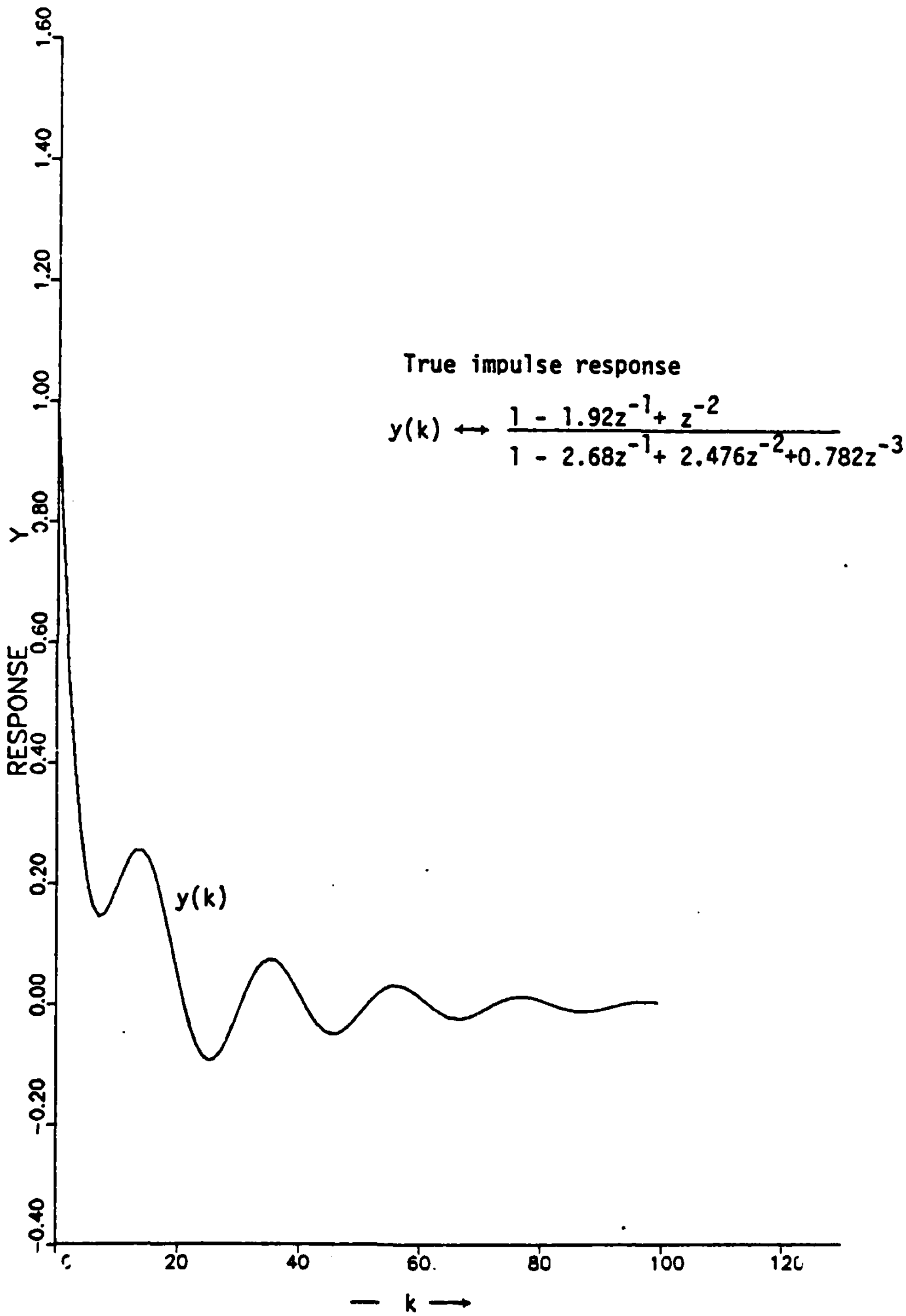


Fig. B1 True impulse response of a third order transfer function

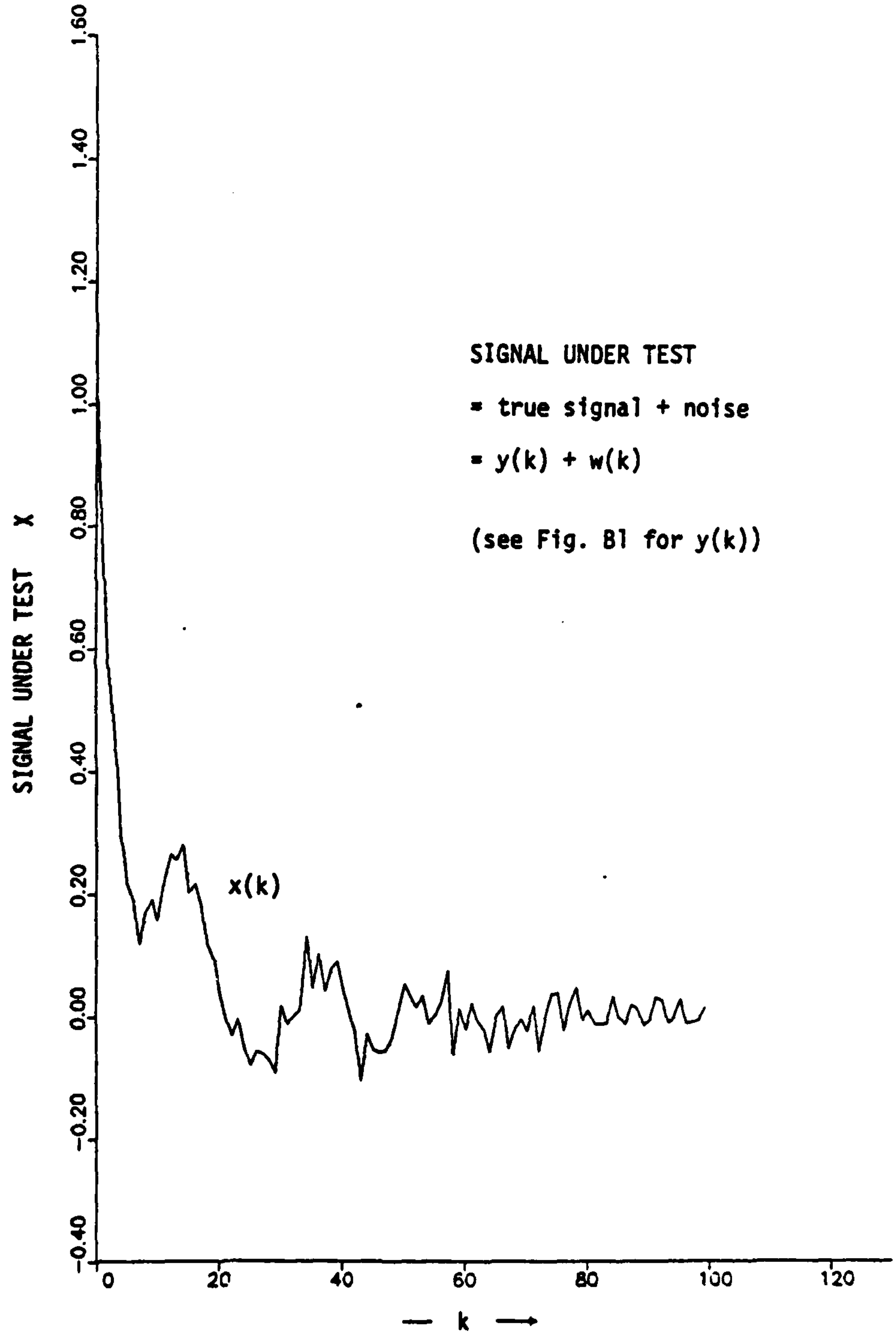


Fig. B2. A simulated noisy signal under test

PRINTER OUTPUT

EXAMPLE 1 LPC. PRONY VS PENCIL-OF-FNS.

```
IPR IFIN ISPN IFIX NFIX    -- PENCIL-OF-FUNCTIONS--
 3-1 1-1-1 1-3 1 100 -3.8
VARIANCE OF NOISE=   .10001-02
FBAR=  .5724E-01 EBAR=  .5354E-02 FL2=  .45571E-01 EE=  .81722E-01
   SYNTHETIC VECTOR, AND SQRT(Y11)
1.0000        .33252        .32575        .45638E-01  127.28
        TRUE GRAM MATRIX (DET=  367.801     )
  1.92921        5.69161        18.1984        60.8396
  5.59161        19.3332        64.3907        215.144
  18.1984        64.3907        233.303        809.572
  60.8396        215.144        809.572        2928.24
WAVEFORMS AND NUMER. SCALED BY XMAX=       1.00000
EST TF B(Z)/A(Z)   (FMAX=  .10000E+01)
     1.00000         -2.63435          2.40737          -.757030
     1.03959         -1.95233          1.00140          0.

SS ERROR=  .470150E-01SS SIGNAL=  2.82039     RATIO=  .166697E-01

ORIGINAL SIGNAL(INCLUDES BIAS,IF ANY)

 1.0000  .7600  .5633  .4032  .2863  .2076  .1627  .1459  .1500  .1696
 .1956  .2221  .2437  .2559  .2563  .2437  .2169  .1833  .1400  .0926
 .0443  .0002 -.0379 -.0669 -.0856 -.0932 -.0901 -.0772 -.0562 -.0339
-.0074  .0164  .0412  .0591  .0707  .0754  .0733  .0649  .0515  .0347
 .0161 -.0025 -.0193 -.0330 -.0426 -.0476 -.0479 -.0437 -.0359 -.0254
-.0133 -.0009  .0106  .0204  .0276  .0319  .0325  .0307  .0261  .0194
 .0115  .0032 -.0045 -.0113 -.0172 -.0206 -.0219 -.0211 -.0185 -.0144
-.0093 -.0037  .0018  .0067  .0106  .0133  .0146  .0145  .0130  .0105
 .0073  .0036 -.0001 -.0036 -.0064 -.0085 -.0096 -.0096 -.0091 -.0076
-.0055 -.0031 -.0005  .0012  .0034  .0054  .0063  .0066  .0063  .0054
IMPL.RESP OF MODEL(INC.B-HAT,IF IBIAS.NE.0)

 1.0349  .7571  .5715  .3979  .2603  .1816  .1335  .1179  .1265  .1505
 .1812  .2109  .2332  .2439  .2405  .2232  .1937  .1540  .1106  .0653
 .0220 -.0130 -.0401 -.0569 -.0633 -.0610 -.0459 -.0321 -.0125  .0075
 .0255  .0396  .0487  .0522  .0503  .0437  .0335  .0212  .0082 -.0040
-.0142 -.0217 -.0259 -.0269 -.0246 -.0200 -.0137 -.0065  .0006  .0069
 .0118  .0150  .0163  .0157  .0135  .0102  .0061  .0019 -.0021 -.0054
-.0073 -.0091 -.0092 -.0084 -.0067 -.0046 -.0021  .0003  .0024  .0040
 .0050  .0054  .0052  .0044  .0033  .0019  .0005 -.0009 -.0019 -.0027
-.0031 -.0031 -.0028 -.0022 -.0015 -.0007  .0001  .0008  .0014  .0017
 .0019  .0017  .0015  .0011  .0006  .0001 -.0003 -.0007 -.0009 -.0010
ERROR=F(K)-FREC(K)

 -.0349 -.0271 -.0117  .0052  .0260  .0260  .0292  .0281  .0242  .0191
 .0144  .0113  .0105  .0121  .0158  .0205  .0251  .0295  .0294  .0273
 .0219  .0133  .0023 -.0100 -.0223 -.0332 -.0415 -.0457 -.0457 -.0414
-.0329 -.0212 -.0075  .0069  .0204  .0317  .0399  .0437  .0433  .0386
 .0303  .0191  .0067 -.0061 -.0180 -.0276 -.0342 -.0372 -.0365 -.0323
-.0252 -.0160 -.0057  .0047  .0141  .0216  .0267  .0289  .0282  .0249
 .0193  .0121  .0044 -.0034 -.0104 -.0160 -.0195 -.0214 -.0209 -.0184
-.0143 -.0091 -.0034  .0023  .0073  .0114  .0141  .0153  .0150  .0132
 .0103  .0067  .0027 -.0014 -.0050 -.0079 -.0097 -.0107 -.0105 -.0095
-.0074 -.0049 -.0021  .0007  .0032  .0052  .0065  .0073  .0072  .0065
```

```
IPP IPLM ISPN IFIX NFIX    -- PENCIL-OF-FUNCTIONS: ENHANCED--
 2-1 1-1 1 1+0 1 100 +0.800000 +0.00011
 VARIANCE OF NOISE=    .1000E-02
 FEAR=  .57241-01 LSAR=  .335EE-02 FEE=   .45571-01 EE=   .51721-01
     SYNTHETIC VECTOR, AND SQRT(Y11)
 1.0000         .59282        .32575        .45632E-01  127.26
           TRUE GRAM MATRIX (DET=  367.001     )
   1.82921        5.69161        18.1964        60.8396
   5.69161        18.9382        64.3907        215.144
   18.1964        64.3907        233.303        809.572
   60.8396        215.144        809.572        2928.24
      100.000          100.000          100.000          100.000
      100.000          272.540          744.170          2029.04
      100.000          744.170          3313.90          12533.0
      100.000          2029.04          12533.0          59255.5
J,I GDET,SUMDET,SI: 1 0     .372+03     .152+04     .66E-03
GDET.DFAC.THR:      .37E+03      .11E-03      .40E-01
J,I  SI,CDET 1 0       .37E+03      .66E-03      .42E+02
J,I GDET,SUMDET,SI: 2 0     .42E+02     .10E+05     .97E-04
J,I  SI,CDET 2 0       .42E+02      .97E-04      .84E+00
J,I GDET.SUMDET,SI: 3 0     .84E+00     .49E+06     .20E-05
J,I  SI,CDET 3 0       .84E-00      .20E-05      .36E-03
 ESTIMATED NOISE VAR=   .756732E-03
     SYNTHETIC VECTOR, AND SQRT(Y11)
 1.0000         .55133        .29774        .40024E-01  117.03
           GEST MATRIX   (DET=  .364275E-03)
  LOCATION/INTEGER=      3
   1.85354        5.61594        18.1227        60.7640
   5.61594        18.7265        63.8275        213.609
   18.1227        63.8275        230.756        800.055
   60.7640        213.609        800.055        2863.40
 WAVEFORMS AND NUMER. SCALED BY XMAX=      1.00000
 EST TF B(Z)/A(Z)  (FMAX=  .100000E+01)
      1.00000          -2.65598          2.49231          -.792556
      .556405          -1.88710          .981501          0.

 SS ERROR=  .475036E-02SS SIGNAL=  2.02039     RATIO=  .156642E-02

 ORIGINAL SIGNAL(INCLUDES BIAS,IF ANY)

  1.0000  .7500  .5605  .4032  .2863  .2070  .1627  .1459  .1506  .1656
  .1956   .2221  .2437  .2559  .2563  .2437  .2165  .1833  .1400  .0926
  .0445   .0002 -.0378 -.0663 -.0856 -.0932 -.0901 -.0776 -.0582 -.0339
 -.0074   .0194  .0412  .0591  .0707  .0754  .0733  .0649  .0515  .0347
  .0161  -.0025 -.0193 -.0330 -.0425 -.0476 -.0479 -.0437 -.0359 -.0254
 -.0139  -.0009  .0105  .0204  .0276  .0318  .0326  .0307  .0261  .0194
  .0115   .0032 -.0048 -.0115 -.0172 -.0206 -.0219 -.0211 -.0185 -.0144
 -.0095  -.0037  .0015  .0067  .0105  .0133  .0146  .0145  .0130  .0105
  .0070   .0036 -.0001 -.0036 -.0064 -.0085 -.0096 -.0095 -.0091 -.0076
 -.0055  -.0031 -.0006  .0017  .0034  .0054  .0063  .0066  .0063  .0054
 IMPULSE RESP OF MODEL(I.E.,-HAT,IF IBIAS.NE.0)

  .9964  .7622  .5700  .4130  .2941  .2105  .1606  .1396  .1410  .1550
  .1744   .2151  .2347  .2554  .2540  .2431  .2159  .1818  .1421  .1000
  .0512   .0055 -.0353 -.0625 -.0813 -.0932 -.0940 -.0703 -.0492 -.0237
  .0033   .0291  .0503  .0683  .0759  .0775  .0720  .0603  .0443  .0249
  .0081  -.0134 -.0284 -.0411 -.0475 -.0484 -.0460 -.0384 -.0276 -.0145
 -.0019   .0110  .0216  .0293  .0337  .0346  .0321  .0267  .0191  .0108
  .0011  -.0075 -.0147 -.0200 -.0229 -.0235 -.0217 -.0180 -.0124 -.0060
 -.0015   .0057  .0102  .0137  .0151  .0151  .0149  .0123  .0087  .0046
```

```
 .0971  .1431  .1833  .2051  .2155  .2110  .1925  .1622  .1231  .0790
 .0339 -.0035 -.0441 -.0726 -.0401 -.0966 -.0930 -.0802 -.0601 -.0354
-.0085  .0174  .0404  .0585  .0702  .0750  .0730  .0647  .0513  .0345
 .0157 -.0026 -.0194 -.0331 -.0427 -.0476 -.0479 -.0436 -.0359 -.0254
-.0133 -.0010  .0106  .0204  .0276  .0317  .0324  .0307  .0261  .0194
 .0115  .0032 -.0043 -.0111 -.0172 -.0206 -.0219 -.0211 -.0185 -.0144
-.0093 -.0037  .0016  .0067  .0106  .0133  .0146  .0145  .0130  .0105
 .0073  .0038 -.0001 -.0036 -.0064 -.0085 -.0096 -.0096 -.0091 -.0076
-.0055 -.0031 -.0006  .0016  .0035  .0054  .0063  .0066  .0063  .0054
```

```
IPR IREM ISPN IFIX NFIX    -- LPC:  COV--
 2-1 5-2-1 143 1 100 +1.0
VARIANCE OF NOISE=    .1000E-02
FBAR=  .5724E-01 LBAR=  .5356E-02 FI2=  .4557E-01 EE=  .5172E-01
         TRUE GRAM MATRIX (DET=  .-15503E-01)
  1.03450        1.05594        1.24678        1.46306
  1.05594        1.35565        1.51621        1.62233
  1.24678        1.51621        1.92593        2.27764
  1.46306        1.62233        2.27764        2.94736
WAVEFORMS AND NUMER. SCALED BY XMAX=        1.00000
EST TF B(Z)/A(Z)   (FMAX=  .10000E+01)
     1.00000          -.653233          -.270334          .116398
     .430511          0.                0.                0.

SS ERROR=  .441646     SS SIGNAL=  2.82039     RATIO=  .156661

ORIGINAL SIGNAL(INCLUDES BIAS,IF ANY)

 1.0000  .7600  .5506  .4032  .2063  .2076  .1627  .1459  .1506  .1696
 .1356  .2221  .2437  .2559  .2563  .2437  .2168  .1833  .1400  .0926
 .0446  .0002 -.0373 -.0663 -.0856 -.0932 -.0901 -.0776 -.0562 -.0339
-.0074  .0194  .0412  .0591  .0707  .0754  .0733  .0649  .0515  .0347
 .0161 -.0025 -.0193 -.0330 -.0426 -.0476 -.0479 -.0437 -.0359 -.0254
-.0133 -.0009  .0105  .0204  .0276  .0316  .0323  .0307  .0261  .0194
 .0115  .0032 -.0045 -.0113 -.0172 -.0206 -.0213 -.0211 -.0185 -.0144
-.0093 -.0037  .0015  .0067  .0106  .0133  .0146  .0145  .0130  .0105
 .0073  .0035 -.0001 -.0036 -.0064 -.0085 -.0096 -.0096 -.0091 -.0076
-.0055 -.0031 -.0006  .0016  .0035  .0054  .0063  .0066  .0063  .0054
IMPL.RESP OF MODEL(INC.B-HAT,IF IBIAS.NE.0)

 .9305  .5373  .6436  .4757  .4179  .3272  .2709  .2160  .1767  .1425
 .1156  .0935  .0755  .0613  .0496  .0402  .0325  .0263  .0213  .0173
 .0140  .0113  .0092  .0074  .0060  .0049  .0039  .0032  .0026  .0021
 .0017  .0014  .0011  .0009  .0007  .0006  .0005  .0004  .0003  .0003
 .0002  .0002  .0001  .0001  .0001  .0001  .0001  .0000  .0000  .0000
 .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000
 .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000
 .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000
 .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000
 .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000
ERROR=F(K)-FPRD(K)

 .0695  .1321 -.0373 -.0765 -.1316 -.1196 -.1032 -.0730 -.0261  .0270
 .0300  .1287  .1673  .1546  .2067  .2036  .1963  .1569  .1137  .0753
 .0304 -.0111 -.0470 -.0744 -.0916 -.0930 -.0941 -.0810 -.0610 -.0360
 .0041  .0173  .0411  .0582  .0710  .0749  .0723  .0645  .0512  .0344
 .0159 -.0010 -.0194 -.0331 -.0427 -.0477 -.0473 -.0432 -.0351 -.0254
 .0174 -.0012  .0107  .0204  .0273  .0317  .0324  .0307  .0261  .0194
```

```
 .0063 -.0037 -.0071 -.0095 -.0109 -.0110 -.0101 -.0083 -.0059 -.0030
-.0001  .0023  .0042  .0059  .0074  .0075  .0069  .0057  .0040  .0020
ERROR=F(K)-FRED(K)

  .0136 -.0022 -.0095 -.0106 -.0073 -.0030  .0021  .0063  .0091  .0100
 .0089  .0070  .0035  .0005 -.0027 -.0054 -.0072 -.0080 -.0081 -.0074
-.0064 -.0053 -.0045 -.0041 -.0042 -.0050 -.0061 -.0078 -.0090 -.0102
-.0109 -.0108 -.0097 -.0078 -.0052 -.0021  .0013  .0046  .0075  .0097
 .0110  .0111  .0101  .0081  .0053  .0019 -.0016 -.0053 -.0083 -.0106
-.0119 -.0120 -.0110 -.0090 -.0061 -.0028  .0007  .0041  .0070  .0092
 .0104  .0105  .0099  .0082  .0059  .0029 -.0002 -.0031 -.0057 -.0076
-.0087 -.0090 -.0085 -.0071 -.0051 -.0028 -.0003  .0022  .0043  .0060
 .0070  .0073  .0069  .0059  .0044  .0025  .0005 -.0015 -.0032 -.0046
-.0054 -.0057 -.0055 -.0049 -.0038 -.0022 -.0006  .0009  .0023  .0034
```

```
IPR IRIM ISPN IFIX RFIX   -- LPC: AUTOCORR--
 2-1 4-3-1 1+0 1 100 +1.0
VARIANCE OF NOISE=   .1000E-02
FBAR=  .5724E-01 LBAR=  .3359E-02 F12=  .4557E-01 FE=  .6172E-01
         TRUE GRAM MATRIX (DET=  4.14352    )
  1.92421      2.27761      1.62222      1.46306
  2.27761      2.94741      2.27766      1.62233
  1.62222      2.27766      2.94740      2.27764
  1.46306      1.62233      2.27764      2.94736
WAVEFORMS AND NUMER. SCALED BY XMAX=    1.00000
EST TF B(Z)/A(Z)  (FMAX=  .10000E-01)
     1.00000          -.731671          -.465046E-01      -.706759E-02
     .984574          0.                0.                0.

SS ERROR=  .407369     SS SIGNAL=  2.82039     RATIO=  .144437

ORIGINAL SIGNAL(INCLUDES BIAS,IF ANY)

 1.0000  .7600  .5604  .4032  .2865  .2076  .1627  .1459  .1506  .1696
 .1955  .2221  .2437  .2559  .2563  .2437  .2183  .1833  .1400  .0926
 .0445  .0002 -.0379 -.0669 -.0856 -.0932 -.0901 -.0773 -.0582 -.0339
-.0074  .0184  .0412  .0591  .0707  .0754  .0733  .0649  .0515  .0347
 .0161 -.0025 -.0193 -.0330 -.0426 -.0476 -.0479 -.0437 -.0359 -.0254
-.0137 -.0009  .0106  .0204  .0276  .0313  .0323  .0307  .0261  .0194
 .0115  .0032 -.0049 -.0118 -.0172 -.0206 -.0219 -.0211 -.0185 -.0144
-.0093 -.0037  .0018  .0067  .0106  .0133  .0146  .0145  .0130  .0105
 .0073  .0036 -.0001 -.0036 -.0064 -.0085 -.0096 -.0096 -.0091 -.0076
-.0055 -.0031 -.0006  .0019  .0039  .0054  .0063  .0066  .0063  .0054
IMPL.RESP OF MODEL(INC.B--AT,IF IBIAS.NE.0)

 .9945  .7235  .5732  .4609  .3697  .2966  .2379  .1907  .1531  .1228
 .0985  .0790  .0634  .0509  .0408  .0327  .0263  .0211  .0169  .0136
 .0109  .0087  .0070  .0056  .0045  .0036  .0029  .0023  .0019  .0015
 .0012  .0010  .0008  .0006  .0005  .0004  .0003  .0003  .0002  .0002
 .0001  .0001  .0001  .0001  .0001  .0000  .0000  .0000  .0000  .0000
 .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000
 .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000
 .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000
 .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000
 .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000
ERROR=F(K)-FRED(K)

  .0054  .0364 -.0124 -.0577 -.0834 -.0[illegible] -.0756 -.0449 -.0083  .04[illegible]
```

```
 .0115   .0032 -.0040 -.0115 -.0172 -.0206 -.0219 -.0211 -.0185 -.0144
-.0093 -.0037  .0015  .0067  .0106  .0133  .0146  .0145  .0130  .0105
 .0073   .0036 -.0001 -.0036 -.0064 -.0095 -.0096 -.0095 -.0091 -.0076
-.0055 -.0031 -.0006  .0015  .0039  .0054  .0063  .0066  .0063  .0054
```

```
IPR IPEM ISPA IFIX NFIX    -- PRONY, AUTOCORR--
 2-1 1-3-1 1+3 1 100 +1.0
VARIANCE OF NOISE =    .1000E-02
FBAR=  .5724E-01 WBAR=  .3356E-02 FLE=  .4557E-01 EE=  .31721-01
        TRUE GRAM MATRIX (DET=  4.14352     )
  1.92921       2.27761       1.82222       1.46306
  2.27761       2.94741       2.27766       1.82233
  1.82222       2.27766       2.94740       2.27764
  1.46306       1.82233       2.27764       2.94736
WAVEFORMS AND NOISE. SCALED BY XMAX=      1.00000
EST TF B(Z)/A(Z)   (FMAX=  .10000E+01)
     1.00000          -.731971        -.465346E-01    -.738759E-02
     .999900           .271633E-01    -.535306E-01    0.

SS ERROR=  .403609     SS SIGNAL=  2.82039     RATIO=  .143132

ORIGINAL SIGNAL(INCLUDES BIAS,IF ANY)

 1.0000  .7000  .5530  .4032  .2863  .2076  .1627  .1459  .1506  .1656
 .1956   .2221  .2437  .2595  .2583  .2437  .2185  .1833  .1400  .0926
 .0441   .0002 -.0373 -.0669 -.0856 -.0932 -.0901 -.0770 -.0562 -.0339
-.0074   .0164  .0412  .0591  .0707  .0754  .0733  .0649  .0515  .0347
 .0161 -.0025 -.0193 -.0330 -.0426 -.0476 -.0479 -.0437 -.0359 -.0254
-.0133 -.0009  .0105  .0204  .0276  .0318  .0325  .0307  .0261  .0194
 .0115   .0032 -.0048 -.0115 -.0172 -.0206 -.0219 -.0211 -.0185 -.0144
-.0093 -.0037  .0015  .0067  .0106  .0133  .0146  .0145  .0130  .0105
 .0073   .0036 -.0001 -.0036 -.0064 -.0095 -.0096 -.0093 -.0091 -.0076
-.0055 -.0031 -.0006  .0015  .0039  .0054  .0063  .0066  .0063  .0054
IMPL.RESP OF MODEL(INC.B-HAT,IF IBIAS.NE.0)

  .9995  .7555  .5434  .4405  .3540  .2835  .2276  .1827  .1466  .1176
 .0943   .0757  .0607  .0487  .0391  .0313  .0251  .0202  .0162  .0130
 .0104   .0084  .0067  .0054  .0043  .0035  .0028  .0022  .0018  .0014
 .0011   .0009  .0007  .0006  .0005  .0004  .0003  .0002  .0002  .0002
 .0001   .0001  .0001  .0001  .0001  .0000  .0000  .0000  .0000  .0000
 .0000   .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000
 .0000   .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000
 .0000   .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000
 .0000   .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000
 .0000   .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000
ERROR=F(K)-FHAT(K)

  .0005  .0011  .0174 -.0373 -.0677 -.0763 -.0651 -.0368  .0041  .0520
 .1013   .1465  .1830  .2073  .2172  .2124  .1937  .1631  .1238  .0796
 .0344 -.0081 -.0445 -.0723 -.0899 -.0966 -.0929 -.0801 -.0600 -.0353
 .0085   .0175  .0405  .0585  .0702  .0751  .0730  .0647  .0513  .0345
 .0159 -.0026 -.0194 -.0780 -.0427 -.0476 -.0479 -.0437 -.0359 -.0254
 .0133 -.0010  .0104  .0204  .0276  .0317  .0325  .0307  .0261  .0194
 .0115   .0031 -.0047 -.0115 -.0172 -.0206 -.0219 -.0211 -.0185 -.0144
 .0093 -.0037  .0014  .0067  .0106  .0133  .0146  .0145  .0130  .0105
 .0073   .0036 -.0001 -.0036 -.0064 -.0095 -.0096 -.0094 -.0091 -.0076
 .0055 -.0031 -.0006  .0015  .0039  .0054  .0063  .0066  .0063  .0054
```

```
IPK IAEM ISPN IFIX NFIX    -- PRONY . COV--
 2-1 1-2-1 1-0 1 100 -1.0
VARIANCE OF NOISE=    .1000E-02
FBAR=  .5724E-01 BBAR=  .3356E-02 FL2=  .4557E-01 EL=  .5172E-01
       TRUE GRAM MATRIX (DET=  .21550E-01)
  1.03400      1.08594      1.24678      1.46306
  1.08594      1.35965      1.51621      1.62233
  1.24678      1.51621      1.92893      2.27764
  1.46306      1.62233      2.27764      2.94736
WAVEFORMS AND NUMER. SCALED BY XMAX=      1.00000
EST TF B(Z)/A(Z)   (FMAX=  .10000E-01)
    1.00000         -.653233         -.270334         .116395
    1.00147          .88072E-01      -.213650         0.

SS ERROR=  .39404      SS SIGNAL=  2.82039     RATIO=  .139714

ORIGINAL SIGNAL(INCLUDES BIAS,IF ANY)

 1.0000  .7500  .5606  .4032  .2963  .2076  .1627  .1459  .1506  .1696
 .1956  .2221  .2437  .2559  .2563  .2437  .2183  .1830  .1400  .0926
 .0447  .0002 -.0375 -.0569 -.0696 -.0932 -.0901 -.0776 -.0592 -.0339
-.0074  .0184  .0412  .0591  .0707  .0754  .0733  .0649  .0515  .0347
 .0161 -.0025 -.0193 -.0330 -.0426 -.0476 -.0479 -.0437 -.0359 -.0254
-.0133 -.0009  .0106  .0204  .0276  .0316  .0325  .0307  .0261  .0194
 .0115  .0032 -.0046 -.0117 -.0172 -.0206 -.0219 -.0211 -.0185 -.0144
-.0093 -.0037  .0018  .0067  .0106  .0133  .0146  .0145  .0130  .0105
 .0073  .0036 -.0001 -.0036 -.0064 -.0085 -.0096 -.0097 -.0091 -.0076
-.0055 -.0031 -.0006  .0013  .0030  .0054  .0063  .0066  .0063  .0054
IMPL.RESP OF MODEL(INC.B-MAT,IF ISI-S.NE.0)

 1.0015  .7525  .5433  .4449  .3513  .2859  .2300  .1866  .1500  .1222
 .0989  .0801  .0648  .0525  .0425  .0344  .0278  .0225  .0182  .0148
 .0120  .0097  .0078  .0063  .0051  .0042  .0034  .0027  .0022  .0018
 .0014  .0012  .0009  .0008  .0006  .0005  .0004  .0003  .0003  .0002
 .0002  .0001  .0001  .0001  .0001  .0001  .0000  .0000  .0000  .0000
 .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000
 .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000
 .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000
 .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000
 .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000  .0000
ERROR=F(K)-FREC(K)

 -.0015  .0377  .0125 -.0415 -.0650 -.0784 -.0673 -.0407 -.0002  .0474
 .0967  .1421  .1789  .2033  .2138  .2094  .1910  .1607  .1218  .0778
 .0327 -.0094 -.0457 -.0733 -.0937 -.0973 -.0935 -.0800 -.0594 -.0357
-.0089  .0172  .0403  .0583  .0701  .0749  .0729  .0646  .0513  .0344
 .0159 -.0026 -.0194 -.0331 -.0427 -.0477 -.0479 -.0438 -.0359 -.0254
-.0133 -.0010  .0105  .0204  .0276  .0317  .0325  .0307  .0261  .0194
 .0115  .0032 -.0046 -.0117 -.0172 -.0206 -.0219 -.0211 -.0185 -.0144
-.0093 -.0037  .0018  .0067  .0106  .0133  .0146  .0145  .0130  .0105
 .0073  .0036 -.0001 -.0036 -.0064 -.0085 -.0096 -.0097 -.0091 -.0076
-.0055 -.0031 -.0006  .0014  .0034  .0054  .0063  .0066  .0063  .0054
```

APPENDIX C

MODELING OF A SCATTERER RESPONSE

The signal of interest is the recorded response of a conducting pipe, considered in Example 2 of Section IV. On site digital sampling of the response was carried out as follows:

k=0 to k=300	Sampling interval	= 0.390625 ns
k=301 to 302	"	= 0.703125 ns
k=303 to end	"	= 1.953125 ns

For analysis purposes we resampled the first 300 data points by picking up every 5th point; the remaining data were used as such to gather a total of 245 data points. Note that we have ignored the intermediate sampling rate of the original data points 301-302.

The reconstituted signal was shown in Fig. 12 by the solid line.

Two runs will be presented below. First pertaining to the signal obtained above (from original data). The second pertains to differentiated (actually, differenced) signal produced from the resampled response (Fig. 13).

Given below are the (card deck) input to program POF-FILTER and, suceeding it, the printer output from the program; first for the signal itself and next for the differentiated signal.

<u>INPUT CARDS</u> (RESAMPLED SIGNAL)

```
RESPONSE OF A SCATTERER.
NPT       IFISP IPLT   XMSE                          BIAS
       NCOMP NOPT                  DT                              ANBIAS
0245      8 0+0 1 245 +5.000000 +0.100000 +0.000000 +.44
0.000000 .037201 .0273[illegible] .055[illegible]57 .0[illegible]552 .113956 .13[illegible] .1563[illegible]7 .157316 .170[illegible]
 .165[illegible]7 .153[illegible] .1[illegible]525 .117201 .0[illegible]7[illegible] .073[illegible] .0[illegible] .031[illegible] .0107[illegible]-.009301
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
```

```
.009-10 .00727- .001025-.002541-.009937-.013776-.021057-.026912-.036960-.042057
-.045791-.054524-.057051-.062057-.065308-.067045-.069220-.070571-.071969-.068035
-.068508-.066055-.064110-.062122-.060119-.057015-.054901-.051047-.049735-.044460
-.041290-.037023-.031666-.027377-.025226-.019036-.017660-.015473-.013276-.013214
-.013135-.013041-.015030-.016035-.016363-.017074-.015765-.021795-.024600-.027760
-.032003-.035002-.040030-.042626-.046591-.050447-.052161-.054045-.055551-.057223
-.057020-.059400-.060047-.060519-.060104-.059591-.059073-.056552-.055000-.054204
-.051600-.051071-.049357-.046777-.043003-.040317-.037023-.036016-.033325-.030648
-.028094-.024235-.022652-.019926-.018409-.016034-.016336-.015035-.016400-.015906
-.015404-.014901-.014356-.016036-.017610-.018234-.019069-.020414-.019005-.021502
-.022044-.024741-.024202-.025520-.026371-.026913-.027457-.025904-.027403-.026053
-.026463-.025069-.023330-.022073-.020275-.019545-.017345-.019030-.018642-.016113
-.014692-.012208-.009755-.008595-.005900-.003606-.002516 .000029 .000189 .001405
.002596 .002655 .004919 .006042 .006066 .004595 .004902 .004952 .004906 .004045
.004709 .003601 .003494 .001216 .000017-.001197-.002432-.001509-.002231-.003232
IPR IPEM ISPN IFIX NFIX IBIAS IBO MNPT  -PENCIL OF FUNCTIONS, ENHANCED-
 000 0-1 1 1+1 0 000 +0.600000 +0.00011
```

PRINTER OUTPUT (CONDUCTING PIPE RESPONSE ANALYSIS)

```
RESPONSE OF A SCATTERER.
 IPR IPEM ISPN IFIX NFIX IBIAS IBO MNPT  -PENCIL OF FUNCTIONS, ENHANCED
  000 0-1 1 1+1 0 000 +0.600000 +0.00011
          TRUE GRAM MATRIX (DET=  .351620E-51)
J,I GDET.SUMDET,SI: 1 0     .35E-51     .29E+05     .34E-04
GDET.DFAC,T+R:      .35E-51     .11E-06     .39E-55
J.I  SI,CDET 1 0      .35E-51     .34E-04     .22E-52
J,I GDET.SUMDET,SI: 2 0     .22E-52     .40E+06     .25E-05
J,I  SI,CDET 2 0      .22E-52     .25E-05     .12E-54
J,I GDET.SUMDET,SI: 3 0     .12E-54     .77E+08     .13E-07
J,I  SI.CDET 3 0      .12E-54     .13E-07     .39E-57
 ESTIMATED NOISE VAR=  .304695E-04
          GEST MATRIX   (DET=  .391860E-57)
 EST TF B(Z)/A(Z)  (FMAX=  .170500+00)
    1.00000          -7.60967          25.5007          -49.2445          59.9213
  -47.0545           23.2899          -6.64020          .830051
     0.              .514727E-01     -.252311          .323601          .519693E-01
  -.544243           .579150         -.267669          .423021E-01
 ESTIMATED MEAN=  -.284251-01

 SS ERROR=  .847-2PE-03SS SIGNAL=  .67805+L-01RATIO=  .125060E-01

S-POLES       SR            SI            SMAG          FR         IZTS= 0
   1      -16.5472       88.9815       90.5070       14.4045
   2      -16.5472      -88.9815       90.5070       14.4045
   3       -2.5432      -12.6372       12.8906        2.0516
   4       -2.5432       12.6372       12.8906        2.0516
   5      -22.4736     -213.2003      219.3543       34.9113
   6      -22.4736      213.2003      219.3543       34.9113
   7       -4.2799      -67.6859       67.8211       10.7941
   8       -4.2799       67.6859       67.8211       10.7941
S-DOMAIN DENOMINATOR
  .637930-01  .122300+02  .112080+05  .463320+02  .102090+05  .234040+02
  .242100+02  .179170+01  .100000+01
S-DOMAIN NUMERATOR
 -.773600+02 -.159430+03 -.486470+03 -.432210+03 -.297760+03 -.446010+02
 -.103540+02  .479350+00 -.576000-01
```

INPUT CARDS (DIFFERENTIATED SIGNAL)

```
DIFFERENTIATED RESPONSE OF A SCATTERER.
      IRESP IPLT    XMSE                         BIAS
    N   +NCOMP NRPT               DT                          ANBIAS
5       3 0+3 1 256 +5.000000 +.1953125 +0.000000 +4.
30000 .322010 .306480 .300000 .286350 .294040 .257300 .166530 .109080 .031830
45000-.120370-.173750-.193240-.234200-.281030-.216790-.170310-.211780-.197910
29540-.034250-.119990-.130140-.079550-.049660-.066450-.037430-.053470-.031000
24030 .067400 .121030 .152180 .162060 .202070 .177390 .175100 .172760 .170290
35190 .111770 .099330 .097790 .052940 .041210 .019300 .029450 .051040-.004340
47910-.059040-.090030-.099760-.142200-.140190-.116510-.125670-.167750-.143560
19760-.114300-.085450-.085520-.063590-.065100-.063420-.041930-.020520-.031300
00730-.010230 .032800 .032320 .053440 .053050 .063360 .073760 .062460 .072890
61630 .072100 .050060 .071290 .070740 .048870 .091500 .069650 .026210 .036700
36320 .014720-.007010-.007120-.007180-.023420-.039450-.039510-.050180-.017730
60740-.029360-.068400-.039590-.073960-.038410-.081190-.070150-.030680-.059770
69340-.047330-.025870-.057060-.025030-.024050-.013750-.013510-.013330 .030040
19450 .034740 .019080 .019960 .020030 .031040 .020340 .031340 .020640 .053150
31790 .042700 .043540 .042390 .021510 .053900 .021580 .021950 .021970 .000620
00790 .000940-.020390-.039550-.004250-.039110-.004910-.030300-.030050-.029650
51000-.039940-.009480-.017960-.029250-.039960-.017140-.027820-.006080-.016720
05970-.005[illegible]-.019470-.005860 .005110 .005130 .005180 .005210 .026720 .015960
26540 .005290 .026940 .016100 .037740 .025960 .026940 .016070 .026910 .026770
37540 .026590 .015300 .016660 .015770 .015750 .004900 .005010-.005730 .005020
05620 .005000 .005050-.019430-.016440-.005540-.016350-.005450 .005310-.016190
05420-.028970 .005590-.016200-.005450-.005420-.005440-.005470 .005210-.005700
15900 .004940 .026710 .004650 .025900 .034300 .015000-.006650 .003680 .025290
14210 .024140 .024530 .013570 .024180 .023100 .012640 .023450 .001600 .012160
11910 .000980 .022000 .011230 .000240-.010710-.000130-.000300-.010460-.000610
00760-.011500-.001070-.022760-.012010-.012140-.012350 .070230-.007220-.010010
 IRUN ISRN IFIX NFIX IBIAS IEQ NRPT      - WITH IBIAS= +1,ENHANCED-
0 0-1 1 1+1 1 2+5 +0.700000 +0.00011
```

AD-A092 226 ROCHESTER INST OF TECH NY DEPT OF ELECTRICAL ENGINEERING F/6 9/3
EXTENSION OF PENCIL-OF-FUNCTIONS METHOD TO REVERSE-TIME PROCESS--ETC(U)
AUG 80 V K JAIN, T K SARKAR, D D WEINER N00014-79-C-0598

UNCLASSIFIED TR-80-4 NL

2 of 2

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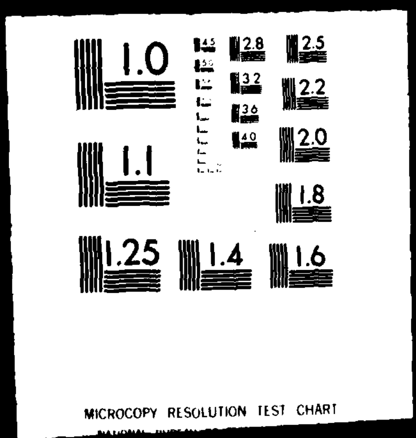
2 OF 2

AD A
092226



MICROCOPY RESOLUTION TEST CHART

PRINTER OUTI

```
DIFFERENTIATED
 IPR IFEM ISPR
  000 0-1 1 1+1
          TRUE
J,I GDET,SUMDET
GDET,DFAC,THR
J,I  SI,CDET 1
J,I CDET,SUMDET
J,I  SI,CDET 2
J,I GDET,SUMDET
J,I  SI,CDET 3
J,I GDET,SUMDET
J,I  SI,CDET 4
J,I GDET,SUMDET
 ESTIMATED NOIS
          GEST
 EST TF G(Z)/A(
    1.00000
  -22.4900
    1.00150
  -3.92049
 ESTIMATED MEAN

 SS ERROR=  .13

S-POLES     SR
   1     -22.267
   2     -22.267
   3      -3.452
   4      -3.452
   5     -25.335
   6     -25.335
   7    -113.302
   8    -113.302
S-DOMAIN DENOMI
  .411710+02  .
  .463270+02  .
S-DOMAIN NUMERA
 -.260910+02 -.
 -.253610+02 -.
```

PRINTER OUTPUT (DIFFERENTIATED SIGNAL)

```
DIFFERENTIATED RESPONSE OF A SCATTERER.

 IPR IPEM ISPN IFIX NFIX IBIAS IBO MAPT      - WITH IBIAS= +1,ENHANCED-
  000 0-1 1 1+1 1 245 +0.700000 +0.00011
         TRUE GRAM MATRIX (DET=  .24993IE-42)
J,I GDET,SUMDET,SI: 1 0      .25E-42      .21E+04      .48E-03
GDET,DFAC,THR:      .25E-42      .11E-03      .27E-46
J,I  SI,CDET 1 0      .25E-42      .48E-03      .76E-43
J,I CDET,SUMDET,SI: 2 0      .76E-43      .23E+04      .35E-03
J,I  SI,CDET 2 0      .76E-43      .35E-03      .24E-43
J,I GDET,SUMDET,SI: 3 0      .24E-43      .41E+04      .25E-03
J,I  SI,CDET 3 0      .24E-43      .25E-03      .72E-44
J,I GDET,SUMDET,SI: 4 0      .72E-44      .56E+04      .17E-03
J,I  SI,CDET 4 0      .72E-44      .17E-03      .22E-44
J,I GDET,SUMDET,SI: 5 0      .22E-44      .56E+04      .12E-03
 ESTIMATED NOISE VAR=  .124001E-02
         GEST MATRIX   (DET=  .63*203E-45)
 EST TF B(Z)/A(Z)   (FMAX=  .33000E-00)
     1.00000        -6.14376        16.9495        -27.6041        30.1560
  -22.4909       11.3523        -3.53420        .512711
     1.00150        -5.19465        12.0095        -16.3922        14.6795
  -5.92049        3.56436        -.803455        .6411638E-01
 ESTIMATED MEAN=   .140352E-01

 SS ERROR=  .130053E-01SS SIGNAL=  .352405     RATIO=  .369044E-01

S-POLES      SR          SI          SMAG          FR         IZTS= 0
   1    -22.2679    -59.2129     63.2616      10.0664
   2    -22.2679     59.2129     63.2616      10.0664
   3     -9.4527    -71.5956     72.2170      11.4937
   4     -9.4527     71.5956     72.2170      11.4937
   5    -25.9956   -222.3405    223.8550      35.6276
   6    -25.9956    222.3405    223.8550      35.6276
   7   -113.3026   -617.0946    627.4099      99.8554
   8   -113.3026    617.0946    627.4099      99.8554
S-DOMAIN DENOMINATOR
  .411710+02  .67**30+02  .217730+03  .171760+03  .264340+03  .638320+02
  .443270+02  .342040+01  .100000+01
S-DOMAIN NUMERATOR
 -.360910+02 -.363640+03 -.512710+03 -.134920+04 -.357040+03 -.223590+03
 -.253610+02 -.431550+01 -.12505C+00
```